THE RECOVERY OF DETAILED MIGRATION
PATTERNS FROM AGGREGATE DATA:
AN ENTROPY MAXIMIZING APPROACH

Frans Willekens

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Preface

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. Recently this interest has given rise to a concentrated research effort focusing on migration dynamics and settlement patterns. Four sub-tasks form the core of this research effort:

I. the study of spatial population dynamics;

II. the definition and elaboration of a new research area called demometrics and its application to migration analysis and spatial population forecasting;

III. the analysis and design of migration and settlement policy;

IV. a comparative study of national migration and settlement patterns and policies.

This paper, the eighth in the comparative study series, addresses the problem of estimating, from aggregate data, detailed migration flows for population categories, such as age and nationality. The estimation procedure proposed is the entropy maximization method. This method has been shown to be useful for a number of countries of the Comparative Study, where disaggregate migration data are not available.

Related papers in the comparative study series, and other publications of the migration and settlement study are listed at the end of this report.

Andrei Rogers
Area Chairman
Human Settlements and Services Area

November 1977.
Abstract

The entropy maximization method is used to estimate inter-regional migration flow matrices for subgroups of the population. The method is presented in lucid terms and a number of practical applications are given, such as the estimation of age-specific migration flows.

Acknowledgments

The practical problem of estimating migration flow matrices by age and by nationality from aggregate data arose in 1974-1975 when I was working with the Brabant Regional Economic Council (GERB-CERB) in Brussels on the development of an urban simulation model for Brussels. I acknowledge with thanks the logistic and financial aid provided at that time by the Council. This paper is a revised version of part of the report on the simulation model (Willekens, 1976).
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vii
The Recovery of Detailed Migration Patterns from Aggregate Data: An Entropy Maximizing Approach

Regional analysis is handicapped by the lack of statistical data. Sophisticated models of regional growth have been developed, but their application is limited since most countries are unable to provide the required input data set. One way to cope with the problem is to estimate or to generate the lacking data. Several estimation methods are known in the literature. One which recently has received considerable attention in urban and regional analysis is entropy maximization.

It is the purpose of this paper to present the method in lucid terms, to allow the practitioner to apply the method in his own situation while realizing the basic assumptions on which the entropy method is based. In the first section, we state the problem for which entropy maximization is relevant. The method is described in the next section. A third section discusses the validity of the method to recover detailed migration patterns from aggregate data. A fourth section applies the method to two practical problems of migration research in Belgium: the estimation of internal migration flows by age, and by nationality.
1. **Entropy Maximization**

1.1 **The Problem**

Suppose we have a two-region system. The problem is to find the origins and the destinations of the migrants that moved in a certain time period. Suppose that the only information we have is the departures and the arrivals by region. We assume a closed system, i.e., the total number of arrivals equals the total number of departures.

The multiregional system may be represented by the following origin-destination table \(\tilde{M}\). (Figure 1).

<table>
<thead>
<tr>
<th>Destination</th>
<th>Origin</th>
<th>Arrivals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>(m_{11})</td>
<td>(m_{12})</td>
</tr>
<tr>
<td>Region 2</td>
<td>(m_{21})</td>
<td>(m_{22})</td>
</tr>
</tbody>
</table>

\[ m_{.1} = 3 \quad m_{.2} = 3 \quad m_{..} = 6 \]

Figure 1

The Origin-Destination Table \(\tilde{M}\)
where

\[ m_{ij} \] is the number of migrants going from region i to region j, e.g., \( m_{12} \) is the number of people migrating from region 1 to 2 in the unit time interval;

\[ m_{1} = \sum_i m_{i1} \]

\[ m_{2} = \sum_i m_{i2} \]

\[ m_{1} = m_{11} + m_{12} = \sum_i m_{i1} \]

\[ m_{2} = m_{12} + m_{22} = \sum_i m_{i2} \]

\[ m_{1} = \sum_j m_{1j} \]

\[ m_{2} = \sum_j m_{2j} \]

\[ m_{..} = \sum_i m_{i..} = m_{..} = \sum_i \sum_j m_{ij} \]

The row totals and column totals, i.e., the total number of arrivals and departures, are known. The problem is to find the entries \( m_{11}, m_{12}, m_{21}, \) and \( m_{22}, \) such that they add up to the known totals. The constraints imposed on the estimation procedure are therefore:

\[ \sum_j m_{ij} = m_{i..} \quad \text{for } i = 1, 2 \] (1)

[10pt]

\[ \sum_i m_{ij} = m_{..j} \quad \text{for } j = 1, 2 \] (2)
There may be a large number of combinations of entries of $M$ that satisfy the constraints (1) and (2). In the simple illustration of Figure 1, there are three possible arrangements.

\[
M_a = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}, \quad M_b = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}, \quad M_c = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}
\] (3)

Each arrangement of the entries of $M$ is called a macrostate of the system.

1.2 The Idea

The true migration flow is represented by one of the three macrostates $M_a$, $M_b$, or $M_c$. Given the limited information we have about the migration behavior, we don't know which macrostate is the true one. Therefore, we must make a guess. It is here that the entropy method comes in. It selects the macrostate which has the highest probability of occurring. A certain macrostate may be generated by various so-called microstates. A microstate is an assignment of individual migrants to the origin-destination table. Consider, for example, the matrix $M_a$, and denote the individual migrants by $m_1$, $m_2$, $m_3$, $m_4$, $m_5$ and $m_6$. According to $M_a$, three people migrate from region 1 to 1, i.e., move within the region. Out of the six migrants, we can select the three in 20 different ways, as shown in Figure 2.
<table>
<thead>
<tr>
<th></th>
<th>m1</th>
<th>m2</th>
<th>m3</th>
<th>m4</th>
<th>m5</th>
<th>m6</th>
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**Figure 2**

Combinations of three out of six
The possible combinations of three people out of six can easily be computed by the familiar combinatorial formula of statistics:

\[
\frac{6!}{3!(6-3)!}
\]

where \(!\) denotes the factorial operation, e.g.,

\[
6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = \prod_{i=1}^{6} i
\]

Once we have made a selection of three people to constitute \(m_{11}\), we must select one person out of the remaining three to constitute \(m_{12}\). There are only three possible ways of selecting one person out of three, or

\[
\frac{3!}{1!(3-1)!} = 3
\]

Finally, the two remaining individuals constitute \(m_{22}\), since \(m_{21} = 0\). Therefore, the total number of ways of selecting three out of six, and one out of the remaining three, and two out of the remaining two is

\[
\frac{6!}{3!(6-3)!} \times \frac{3!}{1!(3-1)!} \times \frac{2!}{2!} = 20 \times 3 \times 1 = 60
\]
Each of the 60 ways constitutes a separate microstate, or assignment of individuals. In general, the number of ways in which we can select a particular macrostate from the total number of migrants $m_{..}$ is the combinatorial formula:

$$W = \frac{m_{..}!}{m_{11}!m_{12}!m_{21}!m_{22}!}$$

$$W = \frac{m_{..}!}{\prod_{i,j} m_{ij}!}$$

Applying (4), we get $W = 60$ for $\sim_a$, $W = 180$ for $\sim_b$ and $W = 60$ for $\sim_c$. The value $W$ is the number of microstates which give rise to a particular macrostate, and is called the entropy of the macrostate.

The question of which macrostate to choose as the best estimate of the true migration flow, may now be answered. We choose the macrostate with the highest entropy-value. The use of this selection criterion relies on two critical assumptions:

- The probability that a macrostate represents the true migration flow matrix, is proportional to the number of microstates of the system which give rise to this macrostate (entropy), and which satisfy the constraints (1) and (2).
- Each microstate is equally probable.
1.3 The Formal Solution

The first assumption, read in a slightly different way, becomes: the true arrangement of a system is one which maximizes the entropy. This is the second law of thermodynamics. This analogy between the behavior of social and physical systems is not accidental. Several authors have attempted to describe social phenomena by laws from physics (e.g., Isard, 1960). This approach is identified as social physics, which is well known in the early regional science literature. It is, however, unfair to evaluate the application of entropy methods in social sciences only by the physical meaning of the entropy concept.

The use of the entropy concept in social sciences may also be justified by means of information theory (Jaynes, 1957), by means of Bayes' theorem for conditional probabilities (Hyman, 1969), or by means of the maximum likelihood estimators (Evans, 1971). See also Wilson (1970, pp. 1-10).

The estimation problem of finding the most probable migration flow matrix which satisfies the constraints (1) and (2), may now be formulated as follows: find the macrostate with maximum entropy, $W$, subject to constraints (1) and (2). The solution is given by the solution to the mathematical programming problem:

$$\max W = \frac{\text{m}..!}{\prod_{i,j} \text{m}_{ij}!} \quad (5)$$
subject to \[ \sum_j m_{ij} = m_i \text{, for all } i \quad (1) \]

\[ \sum_i m_{ij} = m_j \text{, for all } j \quad (2) \]

Since the maximum of (5) coincides with the maximum of any monotonic function of \( W \), we may replace \( W \) by the Naperian logarithm of \( W (\ln W) \) in the objective function.

\[ \ln W = \ln m_{..!} - \ln \prod_{i,j} m_{ij}! \quad (6) \]

\[ = \ln m_{..!} - \sum_i \sum_j \ln m_{ij}! \quad (7) \]

Function (7) is very complex. To make differentiation of (7) easier, we replace \( \ln m_{ij}! \) by Stirling's approximation:

\[ \ln m_{ij}! = m_{ij} \ln m_{ij} - m_{ij} \]

Since \( \ln m_{..!} \) is a constant, we may write the objective function as

\[ \max \ln \hat{W} = - \sum_i \sum_j \left[ m_{ij} \ln m_{ij} - m_{ij} \right] \]

In most applications of entropy methods, the constraints (1) and (2) are augmented with a budget constraint:

\[ \sum_i \sum_j m_{ij} c_{ij} = c \quad (8) \]
where $c_{ij}$ is the cost of moving from $i$ to $j$, and $C$ is the total budget available to all migrants. We introduce this cost function for completeness, although in the applications $c_{ij}$ is set equal to zero for all $i$ and $j$.

Formally, the entropy problem is:

$$\max \hat{W} = - \sum_i \sum_j \left[ m_{ij} \ln m_{ij} - m_{ij} \right]$$  \hspace{1cm} (9)$$

subject to $\sum_j m_{ij} = m_i$, \hspace{0.5cm} for all $i$  \hspace{1cm} (1)

$$\sum_i m_{ij} = m_j \hspace{0.5cm}, \hspace{0.5cm} for \hspace{0.5cm} all \hspace{0.5cm} j$$  \hspace{1cm} (2)

$$\sum_i \sum_j m_{ij} c_{ij} = C \hspace{1cm} .$$  \hspace{1cm} (8)

The solution to this problem is found by the method of the Lagrange multipliers. The Lagrangean is

$$L = \ln \hat{W} + \sum_i \lambda_i \left( m_i - \sum_j m_{ij} \right) + \sum_j \mu_j \left( m_j - \sum_i m_{ij} \right)$$

$$+ \nu \left( C - \sum_i \sum_j m_{ij} c_{ij} \right) \hspace{1cm} .$$

The necessary conditions for a maximum are

$$\frac{\delta L}{\delta m_{ij}} = 0 = - \ln m_{ij} - \lambda_i - \mu_j - \nu c_{ij} = 0$$  \hspace{1cm} (10)$$

$$\frac{\delta L}{\delta \lambda_i} = 0 = m_i - \sum_j m_{ij} = 0$$  \hspace{1cm} (11)$$
\[
\frac{\delta L}{\delta \mu_j} = 0 = m_j - \sum_i m_{ij} = 0 \tag{12}
\]

\[
\frac{\delta L}{\delta \nu} = 0 = C - \sum_i \sum_j m_{ij} c_{ij} = 0 . \tag{13}
\]

From (10) we have

\[\ln m_{ij} = -\lambda_i - \mu_j - \nu c_{ij}\]

or

\[m_{ij} = e^{-\lambda_i - \mu_j - \nu c_{ij}} . \tag{14}\]

Writing

\[A_i = e^{-\lambda_i / m_i} . \tag{15}\]

\[B_j = e^{-\mu_j / m_j} . \tag{16}\]

gives

\[m_{ij} = A_i B_j m_i m_j e^{-\nu c_{ij}} . \tag{17}\]

The coefficients \(A_i\) and \(B_j\) are called balancing factors. They act as repulsion and attraction forces for generating the migration flows. Substituting (17) in (12) yields

\[\sum_j A_i B_j m_i m_j e^{-\nu c_{ij}} = m_i .\]

\[A_i m_i \sum_j B_j m_j e^{-\nu c_{ij}} = m_i .\]
\[ A_i = \frac{1}{\sum_j B_j m_{i,j} e^{-\nu_{Cij}}} \]  \hspace{1cm} (18)

Substituting (17) in (13) gives

\[ \sum_i A_i B_j m_{i,m,j} e^{-\nu_{Cij}} = m_{j,m,j} \]

\[ B_j = \frac{1}{\sum_i A_i m_{i,m,j} e^{-\nu_{Cij}}} \]  \hspace{1cm} (19)

The entropy maximization is then given by the solution of the system of nonlinear equations (17), (18) and (19). Since we have no information on the cost of migrating and on the total budget available, we may assume that \( C_{ij} = 0 \) for all \( i \) and \( j \). The system of equations becomes then:

\[ m_{ij} = A_i B_j m_{i,m,j}, \text{ for all } i \text{ and } j \]  \hspace{1cm} (20)

\[ A_i = \left[ \sum_j B_j m_{j,m,j} \right]^{-1}, \text{ for all } i \]  \hspace{1cm} (21)

\[ B_j = \left[ \sum_i A_i m_{i,m,j} \right]^{-1}, \text{ for all } j \]  \hspace{1cm} (22)

This system has a very simple solution; namely (Raquillet and Willekens, 1977),

\[ m_{ij} = \frac{m_{i,m,j}}{m_{m,j}} \]
Application of this formula to the data in Figure 1 yields:

\[
\begin{bmatrix}
3.4/6 & 3.2/6 \\
3.4/6 & 3.2/6
\end{bmatrix}
= 
\begin{bmatrix}
2 & 1 \\
2 & 1
\end{bmatrix}
\]

which is \( M_B \), the arrangement with the highest entropy.

The first order conditions (10) to (13) only lead to a local extremum of the objective function (9). However, this local extremum coincides with the global extremum since \( \ln W \) is a strictly concave function.

The entropy solution represents the most probable or most likely flow between each region of origin \( i \) and each region of destination \( j \). This distribution function coincides with the production-attraction constrained gravity model (Wilson, 1970, p.42). Therefore, entropy maximization has been used to provide a theoretical underpinning of the gravity model (Nijkamp, 1975, p.210; Wilson, 1970, pp. 47-49). Nijkamp and Paelinck (1974) have attempted to provide a behavioral interpretation of the entropy and gravity model by formulating the entropy model as a dual geometric programming model. By doing so, the objective function, inherent in the entropy maximizing model, attempts to maximize the imputed net "profits" of a spatial system in terms of the difference between positive and negative interaction stimuli (Nijkamp, 1975, p. 213). We will not elaborate on this theory. The interested reader is referred to the literature.
2. **Disaggregation of Migration Flows**

2.1 **The Problem**

Suppose that we know the migration flow matrix of the total population, and that we are interested in the migration patterns of subsets of the population, e.g., sexes, age groups, nationalities, professional categories, etc. Suppose that we also know the number of arrivals and departures of each subset in each region. The system may be represented in a three-dimensional space (Figure 3). We know the outside of the black box, but not its contents. The total migration from \( i \) to \( j \) is \( m_{ij}(\cdot) \), the number of departures from \( i \) by category \( x \) is \( m_{i.}(x) \) and the number of arrivals in \( j \) is \( m_{.j}(x) \).

2.2 **Solution by the Entropy Method**

The problem is to estimate \( m_{ij}(x) \) given the total flow matrix and the departures and arrivals of each category. Assuming zero cost coefficients, the entropy maximizing model is

\[
- \sum_i \sum_j \sum_x \left[ m_{ij}(x) \ln m_{ij}(x) - m_{ij}(x) \right]
\]

subject to

\[
\sum_i m_{ij}(x) = m_{.j}(x) \quad (24)
\]

\[
\sum_j m_{ij}(x) = m_{i.}(x) \quad (25)
\]

\[
\sum_x m_{ij}(x) = m_{ij}(\cdot) \quad (26)
\]

Application of the Lagrangean multiplier technique leads to the following result:

\[
m_{ij}(x) = A_i(x) B_j(x) m_{i.}(x) m_{.j}(x) T_{ij} \quad (27)
\]
Figure 3

The Origin-Destination Black Box
where $A_i(x)$, $B_j(x)$ and $T_{ij}$ are parameters or balancing factors related to the Lagrange multipliers. To find expressions for the parameters, we substitute (27) in the constraints (24) to (26),

\begin{align}
\sum_i A_i(x)B_j(x)m_i.(x)m_j.(x)T_{ij} &= m_j.(x) \\
B_j(x)m_j.(x)\sum_i A_i(x)m_i.(x)T_{ij} &= m_j.(x) \\
B_j(x) &= \left[\sum_i A_i(x)m_i.(x)T_{ij}\right]^{-1} \\
A_i(x)m_i.(x)\sum_j B_j(x)m_j.(x)T_{ij} &= m_i.(x) \\
A_i(x) &= \left[\sum_j B_j(x)m_j.(x)T_{ij}\right]^{-1} \\
\sum_x A_i(x)B_j(x)m_i.(x)m_j.(x)T_{ij} &= m_{ij}(\cdot) \\
T_{ij} &= m_{ij}(\cdot)\left[\sum_x A_i(x)B_j(x)m_i.(x)m_j.(x)\right]^{-1}.
\end{align}

The system of nonlinear equations (28) to (30) does not have a simple solution as in the two-dimensional case. It must be solved by iteration. The steps involved are:

**STEP 1:** Give $A_i(x)$ and $B_j(x)$ an arbitrary starting value, say $A_i^{(0)}(x) = B_j^{(0)}(x) = 0.1$
\[ T_{ij}^{(1)} = m_{ij}(\cdot) \left[ \sum_x A_i^{(0)}(x) \cdot B_j^{(0)}(x) \cdot m_i.(x) \cdot m_j.(x) \right]^{-1} \]

**STEP 2:**

\[ B_j^{(1)}(x) = \left[ \sum_i A_i^{(0)}(x) \cdot m_i.(x) \cdot T_{ij}^{(1)} \right]^{-1} \]

**STEP 3:**

\[ A_i^{(1)}(x) = \left[ \sum_j B_j^{(1)}(x) \cdot m_j.(x) \cdot T_{ij}^{(1)} \right]^{-1} \]

**STEP 4:**

\[ T_{ij}^{(2)} = m_{ij}(\cdot) \left[ \sum_x A_i^{(1)}(x) \cdot B_j^{(1)}(x) \cdot m_i.(x) \cdot m_j.(x) \right]^{-1} \]

**STEP 5:** Go to Step 2.

The stopping criterion of the algorithm may be a combination of the following inequalities.

\[ T_{ij}^{(n)} - T_{ij}^{(n-1)} < \varepsilon_1 \quad (31) \]

\[ A_i^{(n)} - A_i^{(n-1)} < \varepsilon_2 \quad (32) \]

\[ B_j^{(n)} - B_j^{(n-1)} < \varepsilon_3 \quad (33) \]
The need for iteration in solving the problem described by
the expressions (23) to (26) suggests that it is not a generaliza-
tion of the two-dimensional problem given by (9), (1) and (2).
Instead, the extension of the latter problem to three dimensions
would be:

\[
\max - \sum_{i} \sum_{j} \sum_{x} [m_{ij}(x) \ln m_{ij}(x) - m_{ij}(x)] \tag{34}
\]

subject to

\[
\sum_{j} \sum_{x} m_{ij}(x) = m_{i} . (.) \tag{35}
\]

\[
\sum_{i} \sum_{x} m_{ij}(x) = m_{j} . (.) \tag{36}
\]

\[
\sum_{i} \sum_{j} m_{ij}(x) = m_{..} (x) \tag{37}
\]

It can be shown that the entropy solution to this problem is:

\[
m_{ij}(x) = \frac{m_{i} . (.) m_{j} . (.) m_{..} (x)}{[m_{..} (.)]^2} \tag{38}
\]

The problems (23) - (26) and (34) - (37) differ in the constraints,
i.e. in the known data. The first problem assumes that all sides
of the box Figure 3 are known [matrices \( m_{i} (x) \), \( m_{j} (x) \) and \( m_{j} (x) \)].
In the latter only the axes OA, OB and OC are given [vectors \( m_{i} (.) \),
\( m_{j} (.) \) and \( m_{..} (x) \)]. If no other information on migration would be
available than the total number of arrivals and departures by region,
and the composition of migrant categories for all migrants, then (38)
would give the most probable origin-destination flows by category.

\[^{1}\text{I am grateful to Andras Por for pointing out this extension}
and its solution.\]
2.3 Validity of the Estimation Procedure

To assess the validity of the entropy maximizing method, we disaggregate the 1970 migration flow matrix of the total population of Belgium into male and female migration flows, and compare the estimates with the true values published by the National Institute for Statistics. We consider five regions: Brussels, Flemish Brabant, Walloons Brabant, rest of Flanders, and rest of Wallonia.

The input data are given in Table 1.

Table 1

Internal Migration in Belgium, 1970

<table>
<thead>
<tr>
<th>from</th>
<th>Brussels</th>
<th>Fl. Brab.</th>
<th>W. Brab.</th>
<th>R. Flanders</th>
<th>R. Wallonia</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brussels</td>
<td>67,600</td>
<td>8,376</td>
<td>3,091</td>
<td>5,709</td>
<td>10,230</td>
<td>95,006</td>
</tr>
<tr>
<td>Fl. Brabant</td>
<td>13,753</td>
<td>23,459</td>
<td>927</td>
<td>7,986</td>
<td>1,765</td>
<td>47,890</td>
</tr>
<tr>
<td>W. Brabant</td>
<td>5,551</td>
<td>1,469</td>
<td>6,673</td>
<td>448</td>
<td>3,463</td>
<td>17,604</td>
</tr>
<tr>
<td>R. Flanders</td>
<td>5,407</td>
<td>6,870</td>
<td>342</td>
<td>180,346</td>
<td>4,488</td>
<td>197,453</td>
</tr>
<tr>
<td>R. Wallonia</td>
<td>8,383</td>
<td>1,675</td>
<td>3,047</td>
<td>4,136</td>
<td>163,733</td>
<td>180,974</td>
</tr>
<tr>
<td>Total</td>
<td>100,694</td>
<td>41,849</td>
<td>14,080</td>
<td>198,625</td>
<td>183,679</td>
<td>538,927</td>
</tr>
</tbody>
</table>
b. Regional distribution of departures and arrivals by sex.

<table>
<thead>
<tr>
<th>Region</th>
<th>Departures</th>
<th></th>
<th>Arrivals</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>male</td>
<td>female</td>
<td>male</td>
<td>female</td>
</tr>
<tr>
<td>Brussels</td>
<td>47,395</td>
<td>47,611</td>
<td>50,548</td>
<td>50,146</td>
</tr>
<tr>
<td>Flanders</td>
<td>23,968</td>
<td>23,922</td>
<td>20,902</td>
<td>20,947</td>
</tr>
<tr>
<td>Wallonia</td>
<td>8,795</td>
<td>8,809</td>
<td>7,022</td>
<td>7,058</td>
</tr>
<tr>
<td>R. Flanders</td>
<td>98,602</td>
<td>98,851</td>
<td>98,753</td>
<td>99,872</td>
</tr>
<tr>
<td>Total</td>
<td>268,890</td>
<td>270,037</td>
<td>268,890</td>
<td>270,037</td>
</tr>
</tbody>
</table>

Applying the algorithm described by Step 1 to Step 5, we obtain the following values for the balancing factors:

\[
[T_{ij}] = \begin{bmatrix}
0.001413 & 0.000570 & 0.000626 & 0.000054 & 0.000092 \\
0.000421 & 0.002341 & 0.000399 & 0.000166 & 0.000044 \\
0.000462 & 0.000275 & 0.005384 & 0.000025 & 0.000239 \\
0.000060 & 0.000168 & 0.000026 & 0.000920 & 0.000023 \\
0.000117 & 0.000040 & 0.000214 & 0.000025 & 0.000985
\end{bmatrix}
\]

\[
\{A_i(1)\} = \begin{bmatrix}
0.100707 \\
0.099893 \\
0.100822 \\
0.098492 \\
0.101220
\end{bmatrix} \quad \{A_i(2)\} = \begin{bmatrix}
0.099293 \\
0.100099 \\
0.099185 \\
0.101520 \\
0.098800
\end{bmatrix}
\]
\[
\begin{bmatrix}
0.099230 \\
0.100083 \\
0.099310 \\
0.101872 \\
0.099068
\end{bmatrix}
\]
\[
\begin{bmatrix}
0.100773 \\
0.099917 \\
0.100694 \\
0.098172 \\
0.100935
\end{bmatrix}
\]

Entering the balancing factors in equation (27), yields the solution to the entropy maximization problem. The estimates of the migration flows by sex are given in Table 2. The numbers in parentheses are the observed values of the migrations.

**Table 2**

Internal Migration by Sex in Belgium, 1970

a. Males

<table>
<thead>
<tr>
<th>from</th>
<th>Brussels</th>
<th>Fl. Brab.</th>
<th>W. Brab.</th>
<th>R. Flanders</th>
<th>R. Wallonia</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brussels</td>
<td>33836</td>
<td>4137</td>
<td>1539</td>
<td>2767</td>
<td>5116</td>
<td>47395</td>
</tr>
<tr>
<td></td>
<td>(33970)</td>
<td>(4059)</td>
<td>(1489)</td>
<td>(2833)</td>
<td>(5035)</td>
<td></td>
</tr>
<tr>
<td>Fl. Brabant</td>
<td>6965</td>
<td>11726</td>
<td>467</td>
<td>3917</td>
<td>893</td>
<td>23968</td>
</tr>
<tr>
<td></td>
<td>(6936)</td>
<td>(11798)</td>
<td>(451)</td>
<td>(3871)</td>
<td>(912)</td>
<td></td>
</tr>
<tr>
<td>W. Brabant</td>
<td>2785</td>
<td>727</td>
<td>3330</td>
<td>218</td>
<td>1736</td>
<td>8795</td>
</tr>
<tr>
<td></td>
<td>(2728)</td>
<td>(729)</td>
<td>(3364)</td>
<td>(213)</td>
<td>(1761)</td>
<td></td>
</tr>
<tr>
<td>R. Flanders</td>
<td>2780</td>
<td>3487</td>
<td>175</td>
<td>89854</td>
<td>2306</td>
<td>98602</td>
</tr>
<tr>
<td></td>
<td>(2776)</td>
<td>(3462)</td>
<td>(172)</td>
<td>(89746)</td>
<td>(2446)</td>
<td></td>
</tr>
<tr>
<td>R. Wallonia</td>
<td>4182</td>
<td>825</td>
<td>1512</td>
<td>1998</td>
<td>81614</td>
<td>90130</td>
</tr>
<tr>
<td></td>
<td>(4129)</td>
<td>(854)</td>
<td>(1546)</td>
<td>(2090)</td>
<td>(81511)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>50548</td>
<td>20902</td>
<td>7022</td>
<td>98753</td>
<td>91665</td>
<td>268890</td>
</tr>
</tbody>
</table>
b. Females

<table>
<thead>
<tr>
<th>From</th>
<th>Brussels</th>
<th>Fl. Brabant</th>
<th>W. Brabant</th>
<th>R. Flanders</th>
<th>R. Wallonia</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brussels</td>
<td>33764</td>
<td>4239</td>
<td>1552</td>
<td>2942</td>
<td>5116</td>
<td>47611</td>
</tr>
<tr>
<td></td>
<td>(33621)</td>
<td>(4317)</td>
<td>(1602)</td>
<td>(2876)</td>
<td>(5195)</td>
<td></td>
</tr>
<tr>
<td>Fl. Brabant</td>
<td>6788</td>
<td>11733</td>
<td>460</td>
<td>4069</td>
<td>872</td>
<td>23922</td>
</tr>
<tr>
<td></td>
<td>(6817)</td>
<td>(11661)</td>
<td>(476)</td>
<td>(4115)</td>
<td>(853)</td>
<td></td>
</tr>
<tr>
<td>W. Brabant</td>
<td>2766</td>
<td>742</td>
<td>3343</td>
<td>230</td>
<td>1727</td>
<td>8809</td>
</tr>
<tr>
<td></td>
<td>(2823)</td>
<td>(740)</td>
<td>(3309)</td>
<td>(235)</td>
<td>(1702)</td>
<td></td>
</tr>
<tr>
<td>R. Flanders</td>
<td>2627</td>
<td>3383</td>
<td>167</td>
<td>90492</td>
<td>2182</td>
<td>98851</td>
</tr>
<tr>
<td></td>
<td>(2631)</td>
<td>(3408)</td>
<td>(170)</td>
<td>(90600)</td>
<td>(2042)</td>
<td></td>
</tr>
<tr>
<td>R. Wallonia</td>
<td>4201</td>
<td>850</td>
<td>1535</td>
<td>2138</td>
<td>82119</td>
<td>90844</td>
</tr>
<tr>
<td></td>
<td>(4254)</td>
<td>(821)</td>
<td>(1501)</td>
<td>(2046)</td>
<td>(82222)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>50146</td>
<td>20947</td>
<td>7058</td>
<td>99872</td>
<td>92014</td>
<td>270037</td>
</tr>
</tbody>
</table>

The first observation is that the method tends to underestimate high values of $m_{ij}(x)$, and to overestimate low values. A similar observation has been made by Chilton and Poet (1973, p. 140) and Nijkamp (1975, p. 221). This feature may in part be related to the use of Stirling's approximation. Recall that we have set

$$ \ln m_{ij}! = m_{ij} \ln m_{ij} - m_{ij}. $$

Therefore, the derivative is

$$ \frac{d \ln m_{ij}!}{d m_{ij}} = \ln m_{ij}. $$ (39)
Chilton and Poet (1973, p. 137) show that the maximum error, associated with (3a) is

\[ \ln(m_{ij} + 1) - \ln m_{ij} = \ln \left(1 + \frac{1}{m_{ij}}\right) , \]

which is small providing \( m_{ij} \) is large. However, the error becomes greater as \( m_{ij} \) becomes small. If we get errors associated with small values, then, because of the constraints imposed by the totals, we will get equal and compensating errors in high values.

Another more serious cause of the deviation between the estimated and the observed values is the fact that people do not behave according to the entropy maximizing principle. Random elements cause the behavior to deviate from the most likely one. However, given the limited information we have, it is our best available means for recovering hidden migration patterns. The estimated flow matrix has the highest probability to be the true one.

2.4 Application of the Estimation Procedure

The purpose of presenting the entropy method was to come up with a technique to recover detailed migration patterns from aggregate data. The detailed information we are interested in is the migration behavior of aliens living in Belgium, and the age distribution of the migrants. The estimation of this unknown information is the subject of this section.
A. Estimation of migration pattern by nationality.

Each year, the National Institute for Statistics (N.I.S.) publishes the origin-destination table of migrations by sex between and within the 43 arrondissements\(^1,2\). No information is available on the breakdown by nationality or by age. The published data on the internal migration by nationality are limited to the annual number of arrivals and departures by sex and arrondissement for the aliens as a whole\(^3,4\). The arrivals and departures of the Belgians may then be found by subtracting the aliens from the total population. For the system of five regions, the totals of arrivals and departures by nationality are given in Table 3. The number of departures of the aliens have been adjusted such that their total is equal to the total number of arrivals. The data for Belgians


\(^2\)The intra-arrondissement migrations are between municipalities.

\(^3\)See for example, N.I.S., (1975) pp. 132-133.

\(^4\)Each year there is a fictitious positive net migration for the country as a whole. The reason is that some people leave their municipality without informing the local authorities, but register in the municipality of destination. They are added to the total number of arrivals, but not to the number of departures. For our analysis, we have adjusted the number of departures such that the total net internal migration is zero.
are residuals between total population (Table 1b) and aliens.

Table 3

<table>
<thead>
<tr>
<th>Region</th>
<th>Departures</th>
<th></th>
<th></th>
<th>Arrivals</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aliens</td>
<td>Belgians</td>
<td></td>
<td>Aliens</td>
<td>Belgians</td>
</tr>
<tr>
<td></td>
<td>Published</td>
<td>Adjusted</td>
<td></td>
<td>Published</td>
<td>Adjusted</td>
</tr>
<tr>
<td>Brussels</td>
<td>20,402</td>
<td>20,495</td>
<td>80,199</td>
<td>20,538</td>
<td>74,468</td>
</tr>
<tr>
<td>Fl. Brabant</td>
<td>2,454</td>
<td>2,466</td>
<td>39,383</td>
<td>3,056</td>
<td>44,834</td>
</tr>
<tr>
<td>W. Brabant</td>
<td>1,475</td>
<td>1,482</td>
<td>12,598</td>
<td>1,623</td>
<td>15,981</td>
</tr>
<tr>
<td>R. Flanders</td>
<td>11,133</td>
<td>11,184</td>
<td>187,441</td>
<td>11,139</td>
<td>186,314</td>
</tr>
<tr>
<td>R. Wallonia</td>
<td>27,242</td>
<td>27,366</td>
<td>156,313</td>
<td>26,637</td>
<td>154,337</td>
</tr>
<tr>
<td>Total</td>
<td>62,706</td>
<td>62,993</td>
<td>469,502</td>
<td>62,993</td>
<td>469,502</td>
</tr>
</tbody>
</table>

The migration flows of the total population have been given in Table 1a.

Given the information in Tables 1 and 3, we may apply the entropy method to recover the origin-destination table for aliens and Belgians. The result is given in Table 4.
Table 4

Internal Migration in Belgium by Nationality, 1970

a. Aliens

<table>
<thead>
<tr>
<th>from</th>
<th>Brussels</th>
<th>Fl. Brabant</th>
<th>W. Brabant</th>
<th>R. Flanders</th>
<th>R. Wallonia</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brussels</td>
<td>16,300</td>
<td>919</td>
<td>513</td>
<td>695</td>
<td>2,111</td>
<td>20,538</td>
</tr>
<tr>
<td>Fl. Brabant</td>
<td>1,437</td>
<td>1,015</td>
<td>63</td>
<td>387</td>
<td>154</td>
<td>3,056</td>
</tr>
<tr>
<td>W. Brabant</td>
<td>660</td>
<td>73</td>
<td>520</td>
<td>25</td>
<td>344</td>
<td>1,623</td>
</tr>
<tr>
<td>R. Flanders</td>
<td>624</td>
<td>331</td>
<td>26</td>
<td>9,725</td>
<td>433</td>
<td>11,139</td>
</tr>
<tr>
<td>R. Wallonia</td>
<td>1,474</td>
<td>128</td>
<td>359</td>
<td>352</td>
<td>24,324</td>
<td>26,639</td>
</tr>
<tr>
<td>Total</td>
<td>20,495</td>
<td>2,466</td>
<td>1,482</td>
<td>11,184</td>
<td>27,366</td>
<td>62,993</td>
</tr>
</tbody>
</table>

b. Belgians

<table>
<thead>
<tr>
<th>from</th>
<th>Brussels</th>
<th>Fl. Brabant</th>
<th>W. Brabant</th>
<th>R. Flanders</th>
<th>R. Wallonia</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brussels</td>
<td>51,300</td>
<td>7,457</td>
<td>2,578</td>
<td>5,014</td>
<td>8,119</td>
<td>74,468</td>
</tr>
<tr>
<td>Fl. Brabant</td>
<td>12,316</td>
<td>22,444</td>
<td>864</td>
<td>7,599</td>
<td>1,611</td>
<td>44,834</td>
</tr>
<tr>
<td>W. Brabant</td>
<td>4,891</td>
<td>1,396</td>
<td>6,153</td>
<td>423</td>
<td>3,119</td>
<td>15,981</td>
</tr>
<tr>
<td>R. Flanders</td>
<td>4,783</td>
<td>6,539</td>
<td>316</td>
<td>170,621</td>
<td>4,055</td>
<td>186,314</td>
</tr>
<tr>
<td>R. Wallonia</td>
<td>6,909</td>
<td>1,547</td>
<td>2,688</td>
<td>3,784</td>
<td>139,409</td>
<td>154,337</td>
</tr>
<tr>
<td>Total</td>
<td>80,199</td>
<td>39,383</td>
<td>12,598</td>
<td>187,441</td>
<td>156,313</td>
<td>475,934</td>
</tr>
</tbody>
</table>
B. Estimation of migration pattern by age.

The second application of the entropy maximization is the disaggregation of the total migration flow of Table 1 into age-specific flows. To perform this task, we need to know the age structure of outmigrants and inmigrants in each region, in addition to Table 1. The N.I.S. does not publish the age structure of the migrants. The required input information, however, is provided by Delanghe (1974). He publishes, in percentages, the age structure of the emigrants and the inmigrants, by sex, for the nine Belgian provinces and for Brussels, Flemish Brabant and Walloons Brabant. The statistics are based on a sample of 100 municipalities. In each municipality of the sample, the characteristics of the people who located in the municipality in 1968 were collected from the Family Registration Forms (Delanghe, 1974, p. 6). The total number of inmigrants in the sample was 51,784, or 9.76% of the total number of internal migrants in 1968. Table 5 gives the age structures of the outmigrants and the inmigrants for the five regions under consideration. The percentage distributions are weighted averages of the male and female age structures published by Delanghe (1974, pp. 25-26 BIS). The number of migrants in each age group has been found by applying the percentage distribution to the total number of outmigrants and inmigrants of each region in 1970. These totals are the row and column totals in Table 1 minus the diagonal elements.
### Table 5

**Age Structure of Total Population, Outmigrants and Immigrants, 1970**

<table>
<thead>
<tr>
<th>Age</th>
<th>Population</th>
<th>Departures</th>
<th>Arrivals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>Percent</td>
<td>Absolute</td>
</tr>
<tr>
<td>0</td>
<td>68417.</td>
<td>6.364</td>
<td>3786.</td>
</tr>
<tr>
<td>5</td>
<td>70242.</td>
<td>6.533</td>
<td>2232.</td>
</tr>
<tr>
<td>10</td>
<td>68582.</td>
<td>6.379</td>
<td>1638.</td>
</tr>
<tr>
<td>15</td>
<td>66443.</td>
<td>6.180</td>
<td>1638.</td>
</tr>
<tr>
<td>20</td>
<td>78894.</td>
<td>7.338</td>
<td>5156.</td>
</tr>
<tr>
<td>25</td>
<td>69733.</td>
<td>6.492</td>
<td>4515.</td>
</tr>
<tr>
<td>30</td>
<td>66528.</td>
<td>6.188</td>
<td>2702.</td>
</tr>
<tr>
<td>35</td>
<td>69149.</td>
<td>6.432</td>
<td>2408.</td>
</tr>
<tr>
<td>40</td>
<td>73328.</td>
<td>6.620</td>
<td>1669.</td>
</tr>
<tr>
<td>45</td>
<td>79091.</td>
<td>7.356</td>
<td>1483.</td>
</tr>
<tr>
<td>50</td>
<td>57350.</td>
<td>5.334</td>
<td>803.</td>
</tr>
<tr>
<td>55</td>
<td>57656.</td>
<td>5.293</td>
<td>1239.</td>
</tr>
<tr>
<td>60</td>
<td>66088.</td>
<td>6.147</td>
<td>1064.</td>
</tr>
<tr>
<td>65</td>
<td>60717.</td>
<td>5.647</td>
<td>985.</td>
</tr>
<tr>
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**fl.brab**

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Given Table 5 together with Table 1 we may apply the entropy method to decompose the total migration flow into age-specific flows. Note that we have ignored intra-regional movements, i.e., the diagonal elements of Table 1 are set equal to zero. Therefore, the diagonal elements of the $T_{ij}$ matrix are zero in each iteration. The solution of the entropy maximization is given in Table 6. Rounding errors cause the number of arrivals and the total number of migrants (column totals) to deviate slightly from the data in Tables 1 and 5. The departures are recovered exactly.

3. Conclusion

The objective of this paper was to demonstrate the use of entropy maximization in the estimation of detailed migration patterns. The entropy method has received considerable attention in the regional science literature. This paper reviews the important features of the method and applies it to a practical problem.

The basic idea of the entropy method is that the best estimate is given by the most probable migration pattern. This is the pattern with maximum entropy.

The method is particularly useful to derive origin-destination migration flows for several population categories from data consisting of

i. the flow matrix of the total population (all categories) and

ii. the number of arrivals and departures by region and category (i.e., age, nationality). The entropy method is applied to estimate the origin-destination migration flows by age and
by nationality for a five-region system of Belgium.

The entropy method belongs to a broader class of sum-constrained optimization problems. A comparison with other methods, in particular the RAS method, is given in a forthcoming paper. (Raquillet and Willekens, 1977).
Table 6: Internal Migration by Age, Belgium, 1970

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