

# Modeling Transient States in Language Change

Gertjan Postma  
Meertens Institute Amsterdam

## Abstract

Models of language change may include, apart from an initial state and a terminal state, an intermediate state T. Building further on Postma (2010), who observed that the dynamics of the transient state T ("failed change") has an algebraic linking of the dynamics of the overall change  $A \rightarrow B$ , we present a generalized algebraic model that includes both the failed change  $0 \rightarrow T \rightarrow 0$  and the successful change  $A \rightarrow B$ . As a preparatory step, we generalize the algebraic *function* (logist) of a two-state change  $A \rightarrow B$  to a *differential equation* (DE), which represents the law that rules the change. This DE has a bundle of time shifted logistic curves as its solution. This is identified as Kroch's Constant Rate Hypothesis. By modifying this DE, it is possible to describe the dynamics of the entire  $A \rightarrow T \rightarrow B$  process, i.e. we have a model that includes both the successful and the failed change. The algebraic link between failed change and successful change (the former is the first derivative of the latter) turns out to be an approximation.

## 1. Introduction

Failed changes are often left out of studies of language change. This is understandable as studies of change are inherently *post hoc*: we want to know why and how a certain change has come about. Changes that did **not** occur, do not have such a *post hoc* legitimation, and failed changes, that come and fade away, similarly lack such a *post hoc* legitimation and interest, and are equally ignored in the research. With no good reason: what doesn't succeed and why is very relevant to any comprehensive theory of change. For instance, a failed change  $0 \rightarrow T \rightarrow 0$  can tell us something about the stability of the grammar with respect to the feature  $[\pm \tau]$  that is connected to T.

The relevance of failed changes, however, exceeds this stability issue. Failed changes are important to understand successful changes. This can be seen as follows. Successful changes  $A \rightarrow B$ , going from a stable state A to a stable state B, must have a trigger. Call this trigger T. This trigger T is necessarily temporary: it only exists as long as the transition  $A \rightarrow B$  is underway. To the extent that T is linguistic — and our theory models that case —, T is a *failed change*, and the change  $0 \rightarrow T \rightarrow 0$  must have some causal connection to the overall process  $A \rightarrow B$ . In order to understand the linguistic nature of  $0 \rightarrow T \rightarrow 0$  and its connection i.e. interaction with  $A \rightarrow B$ , our challenge is to model this causal interaction, i.e. one must construe a model that includes A, T and B, i.e. we have to design a three-state model of linguistic change.

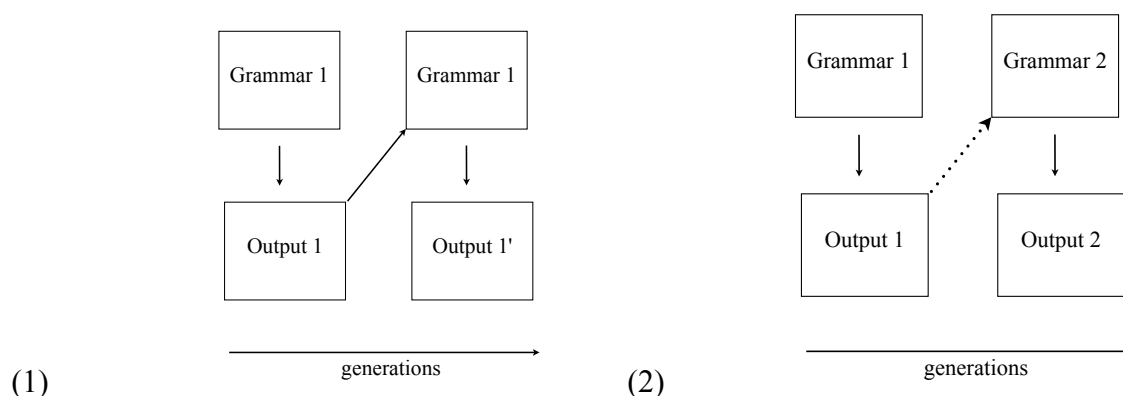
Apart from the standard two-state model of linguistic change,  $A \rightarrow B$ , as discussed in Weinreich *et al.* (1968), Kroch (1989) and others, three-state models has been proposed with an intermediate transient state T:  $A \rightarrow T \rightarrow B$  (Andersen 1973, Van der Wurff 1990, Weerman 1993), where the transient state causes and fuels the transition from one stable state A into a new stable state B. The latter causal models include a linguistic trigger for the change, and these models are theoretically superior to the more descriptive two-state models. However, while the two-state model has received algebraic implementation e.g. the logistic

model (Bailey 1973, Altmann *et al.* 1983), no algebraic implementation of the three-state model has been developed. This is what we are going to do in this paper.

The road map is as follows. Building further on Postma (2010), who observed that the dynamics of the transient state T ("failed change") has an algebraic linking of the dynamics of the overall change  $A \rightarrow B$ , we present a generalized algebraic model that includes both the failed change and the successful change  $A \rightarrow B$ . To extend the logistic model, we must take a step back, or rather, take a meta-position. Instead of taking a parameterized *logistic function* model as basic, we propose that linguistic change  $A \rightarrow B$  is ruled by a *differential equation* (DE) that can be interpreted as the law that governs the change. This DE has a bundle of time shifted logistic curves as its solution. This will be identified as Kroch's Constant Rate Hypothesis. By modifying this DE, it is possible to describe the dynamics of the entire  $A \rightarrow T \rightarrow B$  process, i.e. we have a model that includes the successful and the failed change. The algebraic link, found in Postma (2010) between failed change and successful change (failed change is first derivative of the successful change) turns out to hold by approximation.

## 2. Three-state models

The existing three-state models all implement the idea that natural language should be split in an I-language or Grammar of L, and an E-language, or set of utterances of L, which are generated by L's Grammar (Chomsky 1986). In stable situations without change, the process of language transmission proceeds along the arrows drawn in the figure in (1).

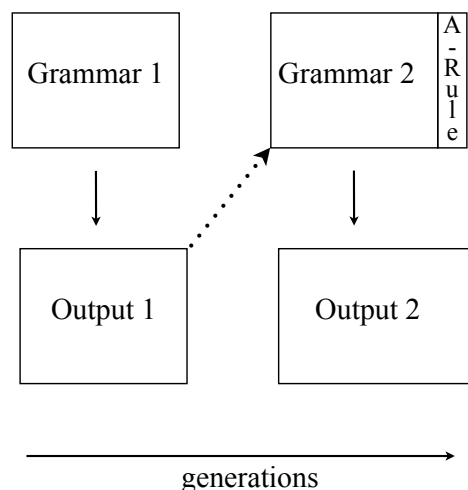


The recognition that the grammars themselves are not transmitted to the next generation is probably Andersen's. A grammar is acquired via its output. The acquisition process of the second generation is based on the set of utterances that the following generation is exposed to. It is in this step where language acquisition can lead to change, if something distorts the perfect learning. This is represented in (2), taken from Andersen (1973:767). The imperfection is represented by the dotted arrow, which causes the new generation to arrive at Grammar 2 instead of the target Grammar 1. Various theories have been proposed about the nature of this imperfect learning, i.e. the nature of the dotted line. We discuss two types here. The first type concerns those theories that situate the imperfection in the learner, i.e. the person that acquires Grammar 2 instead of the target Grammar 1. This type of theory is the oldest and is well expounded in Andersen (1973). The second type of theory situates the change in the adult speakers of the previous generation, who distort their language under sociolinguistic pressure. This is well expounded in Weerman (1993), who is certainly not the first, but who has described it most explicitly. We discuss these below.

### 2.1 Andersen 1973

Andersen's conception of change situates the change in the new generation that acquires the target grammar by trial and error. Those errors are permanently corrected. Andersen draws his

examples from phonology and notes that it is difficult to infer from the perceived *acoustic features* the underlying *articulatory features* in the production. The child of the new generation hypothesizes an (articulatory) grammar and corrects it until it has reached the target Grammar 1. Or not. Failure to duplicate Grammar 1 happens if the child assumes a grammar that approximates Grammar 1, and makes a tentative correction by an Adaptive Rule (or A-Rule as Andersen coins it), probably interpreted as an Adaptive Rule on the output. This is represented in the diagram in (3).



(3)

The quality of Andersen's proposal comes from the insight that Grammars are not transmitted directly but acquired through their output. An secondly, and this feature is not represented in the diagram, the fact that Universal Grammar is active in the acquisition process. Thirdly, there is the attractive feature that the change in grammar is situated in the child, as only children before the maturation age can set formal parameters.

Despite these qualities, at least two objections have been raised to this model. The first objection is formulated by Bybee (1982), who noticed that children before the maturation age do not form communities and are, therefore, unable to transmit their language, i.e. the presupposed accidental "imperfect learning" cannot spread. Only adolescent and adult peer groups can.<sup>1</sup> The second objection emerges from the fact that L1 learners are usually seen as perfect learners. Imperfect learning exists but is situated in L2 learning and other adaptive strategies not executed by the Acquisition Device of language but by some other cognitive device (say, the "Problem Solving Device" of Chomsky (1965). This view complies with the metaphor of children as "little inflection machines" (Wexler 1982:44). This criticism is forwarded most consistently by Weerman's observation that "in 'normal' transmission from generation to generation children are simply too good to be responsible for transmission errors." (Weerman 2011:149). This leads us to the second theory of the source of language change.

## 2.2 Weerman 1993

The other conception of transmission errors situates the source of language change in the adult language use (Van der Wurff 1990, Sankoff & Blondeau 2008), but the most clear advocate is Weerman (1993) who designs a model that we will work out. Weerman tries to

<sup>1</sup> This claim has been challenged, e.g. in Johnson (2007), who argues that children whose parents speak a dialect different from the surrounding community, initially pattern with their parents, but when they start forming a peer group (i.e. once they start schooling) pattern with their peers. This might suggest that children do form communities.

reconcile two or three seemingly incompatible ingredients, mentioned in the previous section. These ingredients are given under (4).

- (4)
- Only children can set linguistic parameters
  - Children are perfect learners and cannot be responsible for transmission errors
  - Children do not have the social infrastructure to spread a change

So, only children can change the language, but they are too good learners to do so. We will call this Weerman's Oxymoron. To resolve this oxymoron of language change, Weerman assumes that adult speakers who have acquired a perfect Grammar 1, embellish it during the lifespan with peripheral rules, which are not part of Grammar 1, and presumably not compatible with any parameterization of UG, for instance, because it is a Output Rule that operates in the postsyntax. They do so under peer group pressure, or accommodation in language contact, or by any fashionable linguistic innovation. When exposed to this embellished grammar, Grammar 1 + Peripheral Rules, the new generation, being perfect learners of UG, (re)set the parameters in order to comply with 1. their parents output and 2. the requirements of UG. As Weerman writes in discussing changes in OV en VO order in the history of English:

(T)hese (VO) leakages were so to speak exaggerated via L2-M acquisition, both quantitatively and qualitatively. The relevant speakers do not change their internalized setting of the head parameter. From the perspective of their L1 grammar these overgeneralizations are ungrammatical. What they do is add a peripheral rule. A next generation, however, could set the head parameter differently". (Weerman 1993)

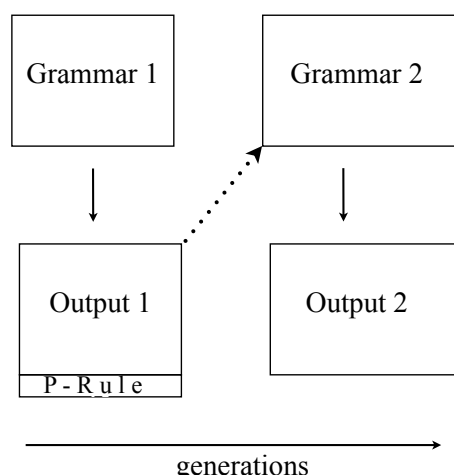
This model with peripheral rules is compatible to suggestions as early as Halle (1962), when he writes:

The language of the adult – and hence also the grammar that he has internalized– need not, however, remain static: it can and does, in fact, change. I conjecture that changes in later life are restricted to the addition of a few rules in the grammar and that elimination of rules and hence a wholesale restructuring of his grammar is beyond the capabilities of the average adult.

The idea is also present in Lightfoot (1999:80):

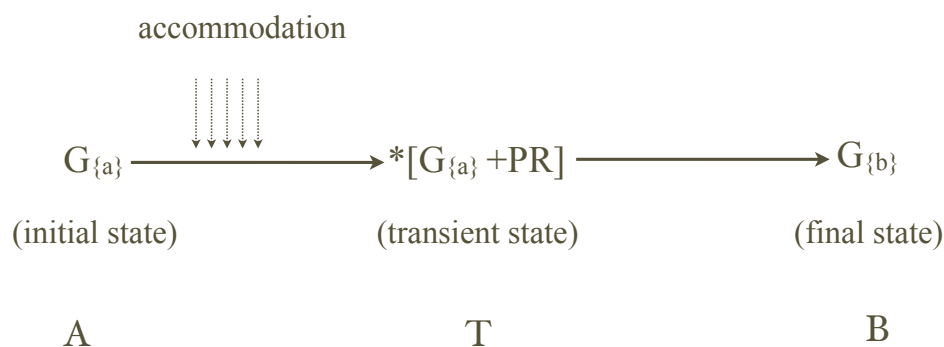
Although adult innovations may not affect grammars, they (...) reflect changes to the primary linguistic data, the input experience for the next generation of language learners. Adult innovations, then, constitute one reason why an individual might be exposed to PLD which differ from what his mother was exposed to.

This model shares the idea of Adaptive Rule and the idea that the grammatical system is active in language change, that we encountered in Andersen (1973). The two theories, however, differ on the locus of these Peripheral (P-Rule) or Adaptive rules, as temporary deviations from true grammaticality. It is given in (5a).



(5a)

Without lack of generality, we can describe this model as a three-state model: a system A, consisting of a pure parameter setting of UG, is temporarily embellished with Peripheral Rules and gets into the excited state T. This situation, which is marked from the perspective of UG, relaxes to a new, pure grammatical state B, produced by a parameter setting of UG.



(5b)

The idea is now that every change passes through a transient state. A transient state is a state that does not fully comply with the principles of UG. This transient state must be seen as the initiator of the change. Moreover, this initiator is also fully part of the change: it is a subpart of the successful change. To see this, it is important to notice that T and B together materialize the change away from A. As to the PLD, T and B are not necessarily different and are, in fact, often indistinguishable. T and B are only essentially different in the way these PLD are *generated*: by  $G_{\{a\}} + PR$  in the case of T; by  $G_{\{b\}}$  in the case of B. But as to their outer form, T and B may be equal or similar: T+B together is the change away from A. Initially, T comprises the majority of the new T+B (=nonA), while in the aftermaths of the change, T comprises only a very small part of T+B (=nonA).

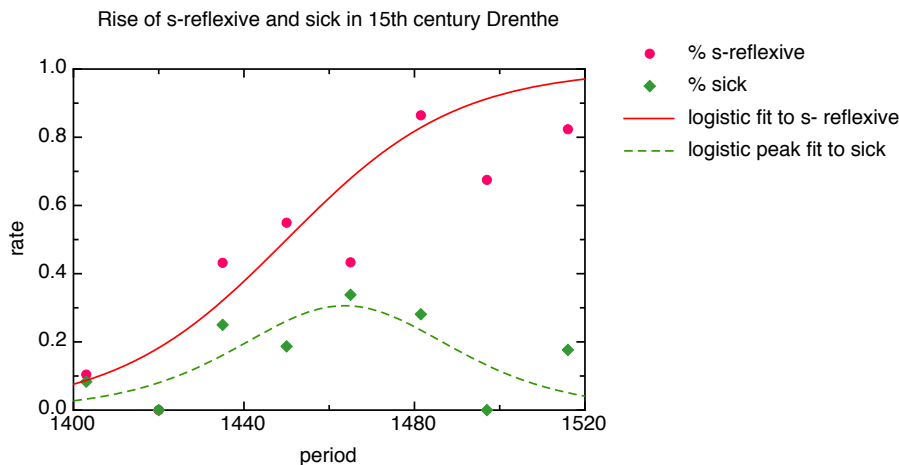
(6) The transient state is a (sub)part of the successful change

In two-state models, T and B are simply collapsed into one. For the purpose of the study of successful changes, T's splitting off from T+B (=nonA) is ignored. For the study of the *causes* of successful changes, however, it is essential to identify T. The fact that T and B are different in the way they are generated, but not essentially different in the way they manifest, makes it notoriously difficult for the researcher to study transient states. In many cases, only highly marked grammatical contexts will tease apart T and B. Only in rare cases are T and B

simply distinguishable at surface level. In the next section, we discuss an instructive case from the literature.

### 2.3 Illustration

An illustrative instance of this three-state change is found in the emergence of the reflexive in Middle Dutch. Middle Dutch had no reflexive pronoun to express contexts with argumental reflexivity, as in 'John saw himself in the mirror', and simply used the pronoun *hem/om* 'him' (as Old English did). In the 15th century, the North Eastern dialects (e.g. in the Drenthe area) began the change. The pronoun *hem/om* 'him' could not be used anymore as a reflexive, and the language borrowed a reflexive pronoun *sick* from a neighboring (Low-German) dialect. However, this pronoun was not well-formed because of phonological reasons, and was gradually replaced by *sich*, borrowed from yet another (Central German) dialect, which completed the change. The curves of the various occurrences are displayed in the figure under (7), taken from Postma (2004). During this century, when *hem/om/...* was gradually replaced by an s-reflexive, we observe a transient period with *sick* that emerges and fades away. These two curves maintain a formal relation to each other, which is interpreted as the sick-line to be the cause of the rise of the s-reflexive. To make the various discussions in this paper more concrete, we will regularly refer to this simple case.



(7)

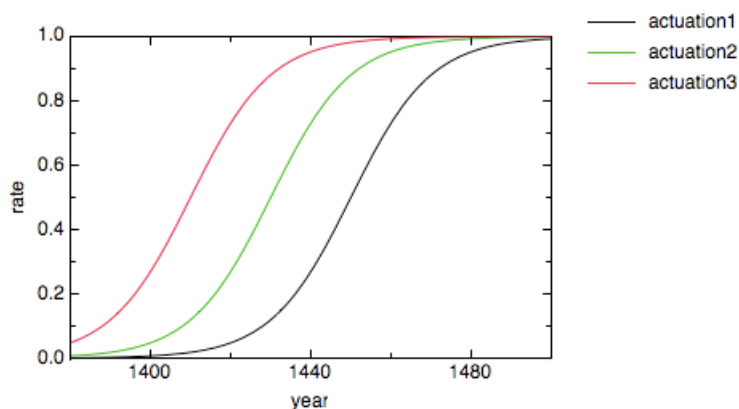
In what follows, we propose an algebraic model that captures the basic ingredients of the three-state model. Before we can do so, we need to briefly recapitulate the two-state model: the logistic model with its advantages and disadvantages. This will be done in the next section.

### 3. The logistic model

The logistic model, proposed in Altmann (1983) and Kroch (1989), is a three-parameter algebraic model that implements the insights in Weinreich *et al.* (1968) quantitatively. While the latter authors take any S-shape curve as the drawn line in (7) as part of their modeling, the former authors specifically opt for the logistic model. The logistic function is given in (8) for further reference.

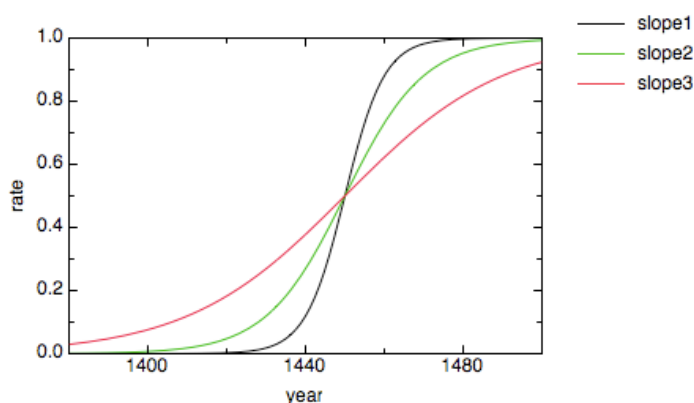
$$(8) \quad S(t) = \frac{S_{max}}{(1 + e^{-\frac{(t-t_0)}{a}})}$$

Kroch's choice for the logist is motivated by pragmatic reasons. It is a widely used model in the life sciences, and it is easy to manipulate graphically: logistic functions turn into straight lines when plotted on bi-logarithmic scales. The logistic function has three parameters: the saturation level  $S_{max}$ , the parameter that situates the change in time  $t_0$ , and the parameter that defines the speed of the change  $a$ . It represents the broadness of the bell curve in The saturation level  $S_{max}$  is usually scalable to 1.<sup>2</sup> We then have two parameters. The parameter that situates the process in time,  $t_0$ , or *actuation time*, is the inflection point in figure (7). On a logarithmic plot, where the S curve is a straight line, it is the *intercept*. Variation in actuation time causes a horizontal shift, illustrated with a linear plot in (9).<sup>3</sup>



(9)

The parameter that defines the speed of the change, or *rate* in Kroch 1989, is the steepness  $a$  of the S curve. On a logarithmic plot it is the slope of the straight line. In (10) we illustrate variation in  $a$  in a linear plot.



(10)

In the next section we discuss these parameters in relation to the Constant Rate Hypothesis.

### 3.1 Kroch's Constant Rate Hypothesis

A change in the grammar does not always show up in the E-language in all contexts at the same time. Usually a change starts in a certain context, and extends its scope to other contexts

<sup>2</sup> By dividing both members by  $S_{max}$ . But this is not always possible, for instance if the change from A to B involves an increase in the use of the construction. In that case, one does not know when the change completes.

<sup>3</sup> Our parameters are natural, i.e. interpretable,  $\alpha$  and  $t_0$ , which are both measured in years/time units. Typical values in (7) are  $\alpha \approx 25$  years for (half of) the broadness of the bell curve, while  $t_0 \approx 1460$ . There is a straightforward conversion to Kroch (1989)'s regression parameters  $s$  (slope), which have 'per time unit' as its dimension and  $k$  (intercept) which is dimensionless. The conversion is:  $t_0 = -k/s$ ,  $\alpha = -1/s$  or  $s = -1/\alpha$ ;  $k = t_0/\alpha$ .

step by step, until it comes to completion. A famous example is *do*-support in the history of English, which starts in negative WH questions and then proceeds to other contexts. Negative declaratives are the last context affected. Kroch (1989) suggests that changes in the E-language that are manifestations of one and the same change in the I-language share a property: the speed with which they proceed (their *rates*) is equal although their actuation times may be distinct. In terms of the logistic function in (8), it turns out that the various curves share the slope parameter  $a$ , while their actuation times  $t_0$  are different. In other words, the various curves in (8/9) may be manifestations of the same change in parameter, while the various curves in (9/10) cannot. This is the Constant Rate Hypothesis (CRH).<sup>4</sup> The CRH has turned out a major tool in the analysis of linguistic change. However, it has not been derived from deeper principles. Let us finish this section with a summary of the advantageous and disadvantages of the logistic model.

(11) *Advantages of Kroch's logistic model*

- Simple logistic two- or three-parameter model
- Well-known from life sciences
- Can be easily displayed and graphically manipulated
- Constant Rate Hypothesis ties E-language process to the I-language (P&P model)

There are also disadvantages of this model. The first disadvantage is the subject of this paper: the relation with transient states under S-curves cannot be made, as was first noticed by Anthony Warner: "(...) the "Constant Rate Effect" (...) (has) the difficulty that the relationship between affirmative declarative *do* and the other contexts is not clear" (Warner 2006:49). This is related to another property of the logistic model: it is a rigid three- or two-parameter model, that cannot be modified, e.g. when we deal with data sets that deviates from this ideal case, nor is it clear under what circumstance it is expected to hold. The disadvantages are listed under (12).

(12) *Disadvantages of Kroch's logistic model*

- Relation with the Transient State is not expressed
- Constant Rate Hypothesis remains a stipulation
- The model cannot be modified
- No instructions are provided under what circumstances it holds

In the next section, we sketch how we can transcend these limitations without losing the advantageous features.

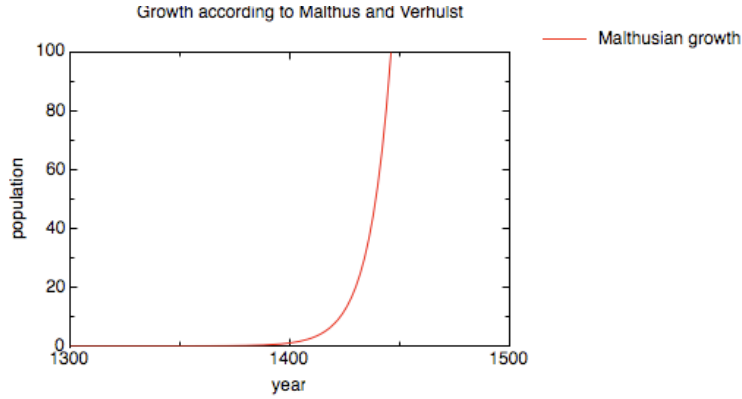
### 3.2 *The underlying Differential Equation*

As said, Kroch's CRH is an empirical generalization. It has not been derived from underlying principles. At this point, it must be noted that logistic curves describe the *kinematics* of a change in the time. It does not describe the actual process behind the change: how do the various players interact dynamically? It may be clear that a description of the outer manifestation cannot give insight in connection between these kinematics. In this section we show how a shift from *functions* (describing the kinematics of a change) to *differential equations*, describing the fundamental interactions, sheds light on the CRH. To illustrate the step, let us first take a simple case of Malthusian growth, as described kinematically by an exponential function under (13).

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<sup>4</sup> The CRH could more transparently be called the Equal Rate Hypothesis. For convenience, we stick to the traditional terminology.





(13)

As Malthus has shown in 1826, the exponential growth is a consequence of a fundamental property of life: the increase over time of a population  $S$  is proportional to the size of the population  $S$ . The bigger the population, the quicker it grows. This can be expressed mathematically by a differential equation: the incremental change in  $S$ ,  $dS$ , over an incremental lapse of time  $dt$ , i.e.  $dS/dt$ , is proportional to  $S$ , as given in (14a). This differential equation has a solution, or rather, a bundle of solutions, as given in (14b).

$$(14) \quad a. \quad \frac{dS}{dt} = aS \quad (\text{Malthus 1826})$$

$$b. \quad S(t) = e^{a \cdot (t-t_0)}$$

This is the exponential function. For every parameter  $a$  in (14a), there is a set of time-shifted solutions, defined by the various  $t_0$ . This model of exponential growth was modified by Verhulst in 1838. Verhulst noticed that there is a counterforce that limits the growth. The more a population grows, the less commodities there are to serve it. Verhulst, therefore, added a factor  $(1-S)$  to the equation in (14a), which reduces the growth when  $S$  becomes bigger. The new differential equation is given in (15a). Its solution is an S-curve, the logistic function. It has, once again, not one solution, but a bundle of time-shifted solutions, defined by the various  $t_0$ . It is given in (15b).

$$(15) \quad a. \quad \frac{dS}{dt} = aS(1 - S) \quad (\text{Verhulst 1838})$$

$$b. \quad S(t) = \frac{1}{(1 + e^{-\frac{(t-t_0)}{a}})}$$

Verhulst's strategy may be clear: DEs have *interpretations*, which are usually rather easy to inspect from their mathematical shape. In our case: the change in  $S$  is proportional to two factors: the available procreators  $S$  and the available commodities. This makes DE's modifiable in a non-arbitrary way. One argues against or in favor of each factor or term in the DE. Subsequently one finds an analytic solution or a solution by computer simulation, rather than modifying the functions directly. The interpretability of the differential equation is essential.

Let us now return to the language change of  $A \rightarrow B$ , which is described by an S-curve (15b). The related DE in (15a) has at least two interpretations that are relevant for the present

discussion. There is the interpretation by Malthus: apart from the factor that stimulate a change, say the wish for offspring/innovation, there is a counterforce, say, food/social acceptability, or whatever. These two factors interact in a certain way. Adding  $(1-S)$  to (14a) leads to (15a) but (14a) can in principle be modified by another type of factor. There is a second interpretation, which is statistical in nature. If  $S$  represents the number of people that have shifted from  $A$  to the innovative  $B$ ,  $1-S$  is the number of people that has not undergone the change. We therefore can give the following interpretation to the underlying DE in (15a). Let us assume that speakers influence each other when they meet. If a  $B$ -speaker meets an  $A$  speaker,  $A$  may change to  $B$ . The increase in the number of speakers  $B$  is proportional to the probability that a speaker of  $B$  meets a person that is still an  $A$ -speaker. If we assume the extremely simple model where all speakers meet each other with equal probability, the DE in (14a) holds.

Notice that the DE in (15a) is the underlying equation that has the logistic function as its solution. Notice further that this solution is not unique but has a bundle of time-shifted solutions. The constant  $a$  in (15b) is fixed by  $a$  in (15a), but  $t_0$  in (15b) is completely free. So, if the equation in (15a) describes a language change, we predict many solutions that instantiate this change. These may have different actuation times,  $t_0$ , but must have equal slopes or rates,  $a$ . This derives Kroch's Constant Rate Hypothesis. We also now know, when we may expect the CRH to hold: only in the case that all (relevant) speakers meet each other by equal chance, i.e. without geographical effects and without diffusion, is (15a) the applicable DE and does the CHR hold. In the next section we apply this promising method to the transient state. We may summarize the conditions of the logistic model and the CRH as in (16).

(16) Conditions of the logistic model

The DE underlying the logistic model (Verhulst's equation) only holds iff

- there are just two variants in competition:  $A$  and not- $A$
- there is only a first-order interaction, i.e. no inherent decline or increase (e.g. increase of literacy might favor a certain construction)
- everyone meets with everyone with equal chance (no geographical effects, diffusion, etc.)

Before we can modify the underlying DE of the logistic model with the presence of a transient state in section 4, we reflect a bit about the interpretability of the models that use Difefretial equations.

*3.3 On the Interpretation of Differential Equations*

The history of differtial equations is intimately linked to the idea of *conservation laws*. In fact, Isaac Newton developed both. Differential equations are supposed to describe the deeper reality modulo the actual realization in space and time. Let me give an example. If we through a ball, we know that it follows a curved trajectory, in fact, we know it follows a parabola. The actual trajectory of the ball is a complex function of the initial conditions of the throwing event and properties of the graviational field, properties of the air, etc. We observe a complex world where everything changes: the position as well as the speed of the ball is different on every moment. There is, however, a deeper reality, which transcents the reality of space and time. In this reality, i.e. the world of energy ( $E$ ) and momentum ( $P$ ), things do not change. The description of the throwing process is much simpler in this energy-momentum space. For instance: Newton taught us that the total energy does not change in time, it is a constant in time. This conservation law is in fact a differential equation. The Newtonian  $E=p^2/2m$  (or the Einsteinian  $E=mc^2$ ), together with the conservation laws, and the energy stored in the ball in

the gravitational field ( $V=gx$ ) provide us with a differential equation,  $(dp/dx)^2=2mgx$ , of which the actual complex trajectory, e.g. the parabola of the Newtonian space, is a *solution*. There are, in fact, infinitely many solutions, an infinite number of trajectories. The boundary conditions of the throwing event selects one of these.

While the trajectories are the visible space, the Newtonian laws represent the interpretable space. We can add terms to the differential equation with specific interpretations: terms that represent the gravitational field, terms that represent frictional terms, etc. In the actual trajectory these terms add up in a complex way. In the trajectories, these contributing factors are not easy to separate. Though the parabola can be considered "a correct model of trajectories of ball throwing", it does not represent a full understanding of what is going on. The differential equations, on the other hand, do offer such interpretive framework. This step from trajectories to differential equation which represent abstract conservation laws is a step from kinematics to dynamics. The later represents the forces that are active in the system at hand.

An anonymous reviewer raises the interesting question how differential equations as mathematical models differ from other statistical models in their interpretability. This is an interesting question of which only a provisional answer can be given. All models must of course have an interpretation, or at least should have, but not all models are equally interpretable, in the sense that one can (easily) assign an interpretation to every subpart. To express this issue in linguistic terms: a kind of composition principle seem to hold in the case of differential equations. A logistic function is more a wholesale parametrized model. Call it 'construction grammar'.

If we apply this way of reasoning to linguistics, we do recognize Kroch's logistic trajectory of linguistic change as a trajectory, but the model does not provide us with an interpretation. As we have seen, it is much easier to give an interpretation to the differential equation of which the logistic is a solution. The DE can be modified with other, equally interpretable terms.

#### 4. The 3-state model

In the previous section we reported a well-established method from physics and the life sciences to modify processes in a non-arbitrary, interpretable way. One adds a well-understood term or factor to the ruling differential equation and solves the new equation analytically or by computer simulation. In this section we apply it to failed changes.

##### 4.1 The failed change model (Postma 2010).

A failed change is defined as a combination of a monotone increasing logist and a subsequent monotone decreasing logist with the same slope. A change is "inherently failing" if the actuation time of the increasing and decreasing logist coincide. If we denote the increasing logist with  $A$ , then the decreasing logist with equal actuation time and opposite slope is  $(1-A)$ . The failed change is therefore  $A(1-A)$ , and by using the DE in (15a), repeated here under 17), we conclude that the failed change  $F=A(1-A)$  is proportional to the first derivative of the successful change.

$$(17) \quad \frac{dA}{dt} = aA(1 - A)$$

So, according to the failed change model, (17) has two interpretations. 1. It can be interpreted as a differential equation that describes the successful change  $A$ . 2. It can be interpreted as a relation between the successful change  $A$  and the failed change defined as  $T=A(1-A)$ . The failed change is the first derivative of the successful change.

$$(18) \quad \frac{dA}{dt} = a.T$$

It has turned out that this model gives quite precise predictions of the transient state that occurs under various successful changes. Not only the example given above of the rise of reflexive in Middle Dutch is nicely described (cf. 6), also the failed change of DO-support in positive affirmative contexts comes out rather precise. The defect of this model is that the failed and successful change are not described by one set of (coupled) DEs, but by one DE (17) which is interpreted in two different ways. In the next section we will correct that by developing the transient state model.

#### 4.2 Preparatory step: transition diagrams

Verhulst's equation describes the transition of  $A \rightarrow B$  by describing the rise of B or the decline of A together with a conservation law of speakers:  $A + B = \text{constant}$ . In the model no persons die, dissolve in the air, or become non-speakers. This can also be expressed by two coupled differential equations as in (19), which is fully equivalent to (17).

$$(19) \quad \frac{dA}{dt} = -a.A.B$$

$$\frac{dB}{dt} = a.A.B$$

The equivalence can be inspected by taking the sum of both members in (19), resulting in  $d(A+B)/dt = 0$ , which states that the change in  $A+B$  is 0, i.e.  $A+B$  is constant.<sup>5</sup> By substituting  $B=1-A$  in the first equation of (19) we derive (17). Similarly we can go from (17) by substitution of  $1-A=B$  in the first equations in (19).

A two-state change  $A \rightarrow B$  can, hence, be expressed by two *coupled* DEs. The equations in (17) have an interesting interpretation. The transition away from A (e.g. the *hem* reflexive) and towards B (s-reflexive) comes about by the interaction of A and B, i.e. the chance that A meets B (i.e. a *hem* speaker meets a *sick/sich* speaker), multiplied by the chance  $a$  that this encounter leads to a change from A (*hem*) to B (*sick/sich*), i.e. the transition coefficient  $a$ . In this case the relation between the two equations in (17) is trivial: the amount by which A (*hem*) diminishes is equal to the amount by which B (*sick/sich*) increases. We can visualize it in the *transition diagram* in (20), as a kind of visual shorthand for (19).

$$(20) \quad \begin{array}{ccc} \text{Transition diagram} & & \text{Transition diagram} \\ A & \xrightarrow{\quad} & B \\ & \text{aA.B} & \end{array} \quad \text{or even simpler} \quad \begin{array}{ccc} \text{Transition diagram} & & \\ A & \xrightarrow{\quad} & B \\ & \text{a} & \end{array}$$

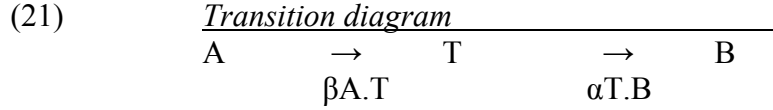
In this model, we have glossed over the distinction of the two s-reflexives. We have collapsed them as representatives of the new s-reflexive but glossed over their different behaviors: *sick* comes up and disappears, while *sich* increases. We equally gloss over the fact that *sick* is initially the dominant form of the s-reflexive, while *sich* is the dominant form of the s-reflexive at the end. In sum, we ignore that *sick* acts as the (ephemeric) trigger of the change.

In the next section, we apply this transition diagram visualisation to the three-state model (i.e. we make a model that takes track of the function of *sick*).

<sup>5</sup> This constant is 1. We always normalize to 1 in this paper.

### 4.3 Modeling transient states

Let us assume a simple stable model without overall increase or decrease in speakers and where children simply replace their parents in society numerically and linguistically. So, if the child enters society, we consider the parent to be removed. This is a rough approximation which excludes interactions between generations in society. By using the techniques developed in the previous section, we can model a three-state change  $A \rightarrow T \rightarrow B$  by assuming the transition diagram in (21).



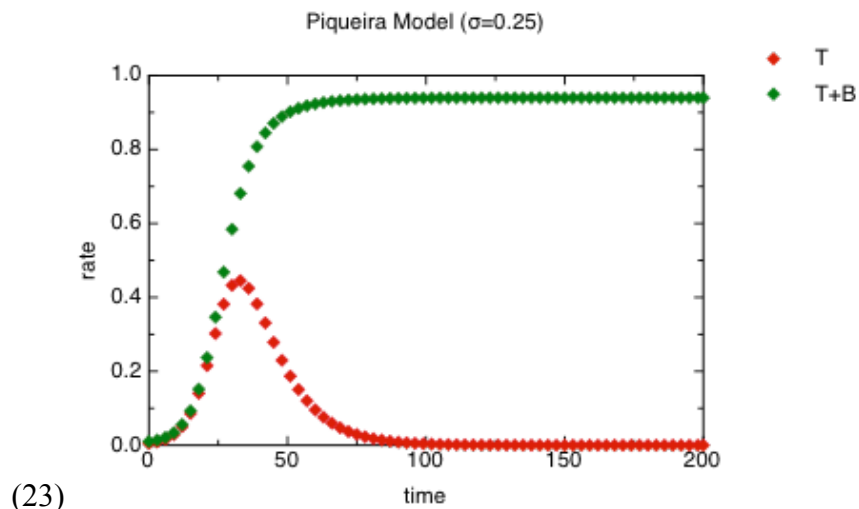
This leads to the coupled differential equations in (22).

$$(22) \quad \begin{aligned} \frac{dA}{dt} &= -\beta A.T \\ \frac{dT}{dt} &= +\beta A.T - \alpha T.T - \alpha T.B \\ \frac{dB}{dt} &= \qquad \qquad +\alpha T.T + \alpha T.B \end{aligned}$$

Let us interpret the terms. The term  $A.T$  is the chance of an encounter of an A-speaker and an innovator T (say the *sick* speaker in our example), which has changed the language with a peripheral rule. If these two speakers meet, speaker A turns in an innovator with chance  $\beta$ . We may call this an L2-interaction, and  $\beta$  the L2 coupling constant. The term  $T.B$  turns an innovator into a B-speaker. This involves a parameter reset, which is only possible upon L1 acquisition, i.e. when the child replaces the adult. One should keep in mind that the innovation T is not really grammatical, as was assumed in the Weerman model, and upon this L1-interaction, the innovator cannot completely resist turning into the more grammatical B (*sich* in the case of the example). It comes about with factor  $\alpha$ . Finally, there is the term  $T.T$  which represents the encounter of an innovator with another innovator. To understand what this means, it is important to notice that we assume that nothing happens whenever A meets A, or whenever B meets B: no terms  $A.A$  or  $B.B$  are included. These are grammatical utterances in a homogeneous speech interaction, which do not cause a change. These are encounters of two *hem*-speakers or two *sich*-speakers: no change happens. This is different, however, in the case of a T-T encounter. It is generally assumed in innovation/rumor theory that innovators stop using the innovation as soon they meet other innovators (Piqueira 2010). -- As if a woman in a Viktor & Rolf dress meets another woman in a similar dress. She will never wear it again. -- In the case of language change we must split this term into an adult-adult interaction and an adult-child interaction (or rather child-adult replacement in our model). The adult-adult interaction does not contribute a change as adults cannot reset parameters, which is needed to switch to B. The adult-child interaction, or rather, when the child replaces the parent in society, results in B. A similar reasoning holds for the interaction between T and B. Switching to B needs a child. So, while the transition coefficient  $\beta$  involves L2-interaction, and the transition coefficient  $\alpha$  involves L1. So we fundamentally have built in the transient intermediate state with its properties into the model. This model has been described and

computer-simulated in Piqueira (2010). It is an implementation of the famous SIR-model<sup>6</sup> in biology modeling, the spreading of diseases (Kermack & McKendrick 1927).<sup>7</sup>

The coupled differential equations in (22) are not solvable by analytic methods. We have solved them using numerical methods.<sup>8</sup> The solution is displayed in the figure in (23).<sup>9</sup>



We see a rather similar picture to the figure in (7), which displays the failed change of *sick* in relation to the successful change of the s-reflexive in the failed change model, where the failed change is the first derivative (or slope function) of the successful change. In the new model, however, this relation is an approximation. Notice that (23) is not a fit to the historical data, which we have not performed yet. It would mean an in tandem solving of differential equations and a fitting process. We hope to carry that out in further research. What the present discussion tells us, is that we succeeded in making a consistent model of a realistic linguistic process that includes the successful and the failed change (transient state), where the transient state is the initiator and driving force of the successful change.

A final comment on (23) is in place. It must be noticed that the T+B line does not reach the full saturation level of 100%. This means that a small number of A-speakers remain after the T-innovation has swept through the community. The reason is that we did not include a term of direct interaction between A and B, say  $\gamma A.B$ . If we add such a term with negative sign to (22a) and with positive sign to (22c), full completion of the change is guaranteed. However, this simply superposes our new model with the terms of the old model (19). These terms are inherently asymmetric. Alternatively, one might add a symmetric change like  $A.B.B \rightarrow B$  and  $A.A.B \rightarrow A$  to the system. This three-person interaction would add an effect of accommodation to the majority. In terms of the equations in (22) it would add a term  $AB(A-B)$  to (22a) and  $AB(B-A)$  to (22c). However, as these changes can only happen within the family (from A to B is a parameter switch), we must assume core families with more than two adults.

<sup>6</sup> Susceptible-Infected-Removed in the epidemiological model.

<sup>7</sup> Piqueira's model is an application of this biological model to the spreading of information: the Daley-Kendall model. The three groups are called Stiflers, Spreaders, and Ignorants. The differential equations in Piqueira's 2.1 and ours (cf. 20) are identical, modulo the error in Piqueira's 2.1).

<sup>8</sup> The Runge-Kutta method. We used the program SolveDiffEq 3.6 for MacOS by Bob Delaney's Science Software 2013.

<sup>9</sup> SolveDiffEq does not have a graphical component. We used ProFit 6.2.11 for MacOS.

## 5. Competing Grammars Approach

The competing-grammars interpretation of the logistic function (Kroch 1989, Yang 2004, Fruehwald & Wallenberg 2013) does not situate the variation found in the E-language as a distribution of A speakers and B speakers (as discrete properties), but as a relative activation of two *grammars* A and B present in each individual. This interpretation can be characterized as an internalization or psychologicalization of the variational space, i.e. as a property of L-language. In the extreme case, it is possible that a completely homogeneous community with an equal A-B grammar contribution in each individual, shifts from the initial value 1 to the final value 0. If we assume that the  $A/A+B$  and  $B/A+B$  ratios represent the chance that, given a particular individual  $x$ , the A grammar or the B grammar is activated in  $x$ , and if we assume that these ratios are equal in all speakers<sup>10</sup>, we end up with a chance that the A grammar in individual 1 'meets' the B grammar in individual 2. We now can simply leave out the level of individuals from the theory and model the encounter of grammar A and grammar B, as if they were meeting directly. Mathematically we end up with the same differential equations and (logistic) solutions. Put differently, this is another *interpretation* of the same mathematical modeling. However, in principle, the competing-grammar interpretation allows for modeling both the individual-internal variational space and the individual external variational space, and probably some interaction between these. This model is therefore potentially a richer model of which the mathematical complexities exceeds the limits of this paper. We therefore limit ourselves to the internalized variational space.

The question is now, if the failed change model has a similar internalized interpretation. This is problematic. While it is perfectly reasonable to assume two grammars in one individual, or that one construction may receive a double analysis in one individual, it is difficult to understand how a specific surface-construction might have both a marked and an unmarked analysis: a marked one with grammar A + peripheral rules, which is transient, and a neutral unmarked analysis that is produced by grammar B. If so, a construction might have a marked, say peer group, reading in grammar A and a completely neutral reading in grammar B. It would be reasonable of optimizing parses that only the most neutral, most optimal parse surface. Moreover, we assumed that re-analyzing construction with the B-grammar was the prerogative of a young pre-maturity individual, while analyzing it with peripheral rules the possibility a post-maturity individual with the A grammar. We, therefore, *must* use the external variational space between the individuals if we want to include failed changes in the competing grammar interpretation. This is not necessarily a disadvantage, as the internal and the external degrees of freedom do not necessarily add independent complexity to the model, and we cannot exclude the external variational space in the competing-grammar approach without ad hoc stipulations anyway.

Finally, we would like to discuss the explanation of Constant Rate Effect in the competing-grammar approach. Notice that the constant rate effect makes crucial use of variation in the intercept parameter of the logistic, which we interpreted as the actuation time according to standard assumptions. Interestingly, Kroch (1987) proposes a different interpretation of this intercept parameter, which is crucial in his explanation of the Constant Rate Effect. For Kroch, the intercept does not reflect the different actuation points for different contexts, but instead the *differential advantage* of the two forms in various contexts. Thus, some contexts will favour the incoming variant, which will be used more frequently in that context at all time periods, while others will favour the outgoing variant. This is a crucial part of Kroch's theory that only one change is actually occurring with one actuation, but with distinct *advantages*. The change actuates only once, and then progresses at the same rate

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<sup>10</sup> We ignore that fact that it is not a logical necessity that the activation of grammars, A or B, be equal in speakers and hearers.

through all the contexts. Different contexts, however, can have preferences for one form which impacts the E-language distribution. Thus, Kroch argues that since there is only one change, it can have only one slope.

This intercept implementation of the "advantages" is not compulsory, however. It is just a possibility to implement the "differential advantages and context favorings" by varying the intercept, as Kroch opts for. It is equally possible to implement the "differential advantage" by varying the rate. In fact, the first option is problematic. In the first place, varying the intercept varies the actuation time, and because of dimensional reasons alone, one cannot hold it directly responsible for a favoring upstep. The relation is indirect by the monotone relation between actuation time and occurrence threshold. One can only *calculate* a corresponding threshold by using the monotone-increasing relation between both given by the logistic function. Moreover, in Kroch's interpretation the "favoring" by certain contexts only happens once at the actuation time, where some contexts get a favoring threshold, an upstep at the very beginning, from which point on, all contexts are treated equally. It is not clear, however, why this "favoring", "advantage" or "upstep" would not be present at intermediate times, or infinitesimally at all times. This would in fact change the rate.

But the major argument against the threshold interpretation is empirical. There is no indication that some contexts start out with a threshold in the E-language. All observed curves are simply complete logistic curves that are slightly shifted with respect to each other. An obvious objection against this counter-argument is that the threshold occurs within the individual speaker, in the internal dynamics of the change, not in the data of the E-language. This objection is legitimate but shows that the competing-grammar interpretation crucially uses both the internal and external variational space. If this model is mathematically expounded in full detail, it will be have a complexity of which our modeling is only a simple sub-case.

## 6. Conclusions

It is possible to generalize the logistic model of language change from a kinematic modeling of the change to a dynamic differential equation that rules the change: the differential equation captures the respective interactions and functions as a process *law*. In the simplest case, this law takes the shape of Verhulst's equation. The solution of this equation is a logistic function, or rather, a bundle of the logistic functions, which are time-shifted with respect to each other. This derives the Constant Rate Hypothesis. By modifying Verhulst's equation with an intermediate state, we made a unified model (coupled differential equations) that models both the successful change and the failed change within one model. The failed change is the first derivative of the successful change, by approximation only.

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