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Spatial demography: A unifying core and agenda for further research

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Abstract

With increases in the availability of geo-referenced data, there has been a push for developing better methods to study demographic processes across space. This paper reviews the recent developments in "spatial demography" and argues that an important aspect has been neglected, namely, the focus on the dynamics and interactions of population change across space, which is an area that should be central to the field. Frameworks for analysing spatial demography were first proposed in multiregional demography. This paper revisits these methods and then describes how methods developed by geographers, economists, and other social scientists for analysing spatial data may be better integrated to study spatial population dynamics.

KEYWORDS

multiregional demography, population geography, spatial analysis, spatial demography

1 | INTRODUCTION

Geo-referenced data on populations and demographic processes are now widely available. As a result, demographers, geographers, economists, and other social scientists have been exploring spatial patterns of these data to better understand population change. We review the recent developments in the field of spatial demography and argue that it is currently missing an important unifying framework that was first proposed in the late 1960s but apparently forgotten (Bell, 2015).

In the past 15 years, there have been several prominent special issues and papers focusing on spatial demography (i.e., Matthews, Janelle, & Goodchild, 2011; Matthews & Parker, 2013; Voss, 2007; Wachter, 2005). The articles included in these collections have largely consisted of spatial analyses of geo-referenced demographic indicators and analyses of migration, as well as techniques for mapping or dealing with spatial autocorrelation in regression models. Traditional notions of spatial demography, as the study of the dynamics and processes of spatial population change, are surprisingly missing. This may be due to preferences for broad conceptualisation or to disagreement of what spatial demography represents. It may also be due to the lack of training and courses available to conduct spatial analyses on demographic processes (De Castro, 2007).

For example, in the introduction to the special issue on "spatial demography" published in the Proceedings of the National Academy of Sciences, Wachter (2005, p. 15300) provides the following viewpoint:

> Spatial demography extends across space and time, from the changing face of the everyday world of present experience to glimmerings of our remote origins and interconnections. It brings sciences together: geography and demography, political and social sciences, mathematics, statistics, physics, and biology.

As another example, Weeks (2016: 108) defines spatial demography as simply

> "... the application of spatial concepts and statistics to demographic phenomena."

In this paper, we argue for a more specific conceptualisation of spatial demography as a guiding framework—one that was first proposed in the late 1960s by Rogers and, more recently, described by Sweeney (2011: 1)

> "... focuses on place-dependence, relative location, and interaction to gain insights into population level processes and individual-or household behavior."

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This notion of spatial demography distinguishes the field from the spatial analyses being conducted in other well-recognised fields of population geography, spatial statistics (including spatial econometrics), regional science, and spatial analyses conducted in public health, political science, sociology, and social statistics.

Our main line of argument is as follows. Demography is the study of population, and as such, it places population dynamics at the core of analysis. Central to the field of demography is population composition and the factors that cause the composition to change. Fertility, mortality, and migration are the mechanisms underlying demographic change. They have distinct age- and sex-specific patterns. Consequently, age and sex are key stratification variables that distinguish demography from other disciplines. Because age is determined by the date of birth and people born around the same time period share important collective experiences that influence their demographic behaviour, the birth cohort is also an important stratification variable in demography. Other stratification variables, such as education, marital status, and ethnicity, may be added to account for the effect of population heterogeneity on population dynamics. Therefore, *spatial demography* is the study of how populations and their compositional structures change and interact across space.

The study of geography, on the other hand, utilises place and space as its central and unifying theme. *Population geography*, therefore, is about how location-specific contexts and connectivity across space drive population change. It focuses on the impacts of location-specific qualities on populations and the reasons for the differences found across space. This field also includes techniques for mapping population data and utilising geographic information systems for analysis. Spatial demography and population geography have much in common, but they differ in focus, with spatial demography focusing on population structure and dynamics, and population geography focusing on the influence of spatial location and distribution on populations.

Demographic processes may be viewed from a variety of perspectives. For example, consider the study of fertility. The current spatial demography literature suggests that any study of fertility is, by definition, demographic. We disagree. There are many disciplines interested in fertility. A geographer may be interested in the patterns of fertility and how they vary across space. A public health researcher may be interested in the demand for infertility treatment. And a demographer may be interested in how fertility is related to the population at risk of producing the births and the implications for population change. Perspective is important. If the interest is focused on populations and how they change and interact across space, then it may be said to be demographic. However, if the interest is in the spatial patterns of population change, then it may be said to be geographic.

Consider another example of migration. A geographer would be interested in why migrants from particular origins go to particular destinations. A sociologist might be interested in how migrants are accepted into society. A political scientist might focus on the policies of entry and access to citizenship. And a demographer would think that an out-migrant from one place is an in-migrant to another and that the migration would have implications on the age and sex structures of both populations. These simplified examples are used to illustrate how one activity may be viewed from multiple perspectives. Of course, most disciplines borrow from other disciplines—what

makes them distinctive is the perspective or direction they take in conducting their arguments and lines of research.

In reviewing the recent literature, we find that the current field of spatial demography largely consists of techniques to describe variations across space and methods for dealing with spatial autocorrelation or spatial context, which is really the realm of population geography or spatial statistics (see, e.g., the overview of recent developments by Matthews & Parker, 2013). For spatial demography to be a distinctive field of enquiry, we argue that demography (i.e., the study of population) needs to be at the core of the thinking. Thus, our purpose in this paper is to redirect some of the current methodological developments in spatial analysis of social data towards improving how we understand the dynamics and intersections of population change across space and over time. In other words, we seek to draw researchers interested in demographic change away from focusing primarily on the analysis of the spatial patterns (i.e., realm of geography) towards analysing the underlying dynamics and processes of demographic change across space. Migration is the link between populations across space and those interactions, therefore, represent a key component in understanding spatial population change.

The boundaries of spatial demography do not necessarily need to be fixed, and in this writing, an inflexible and exclusionary paradigm is not proposed. Several scholars may even argue that making that the distinction is unproductive in our common search for knowledge because the mechanisms of demographic change depend strongly on location and spatial connectivity. Our aim is to contribute to a discussion that clarifies the core of demography in the study of population distributed in space. We understand there are many good reasons and benefits for fields of enquiry to overlap and learn from each other. Indeed, we find spatial demographic research in the main journals of, for example, demography, geography, economics, and sociology. We believe that a stronger core that distinguishes spatial demographic theory and analyses from those conducted in other social spatial sciences would enhance the field, and in this critique, we highlight some of the unique lines of enquiry spatial demographers could conduct with such a core.

In our exposition of what spatial demography should be centred on, we focus on the dynamics of population change and how these dynamics affect different populations across space. We first review the current situation of research in journals where spatial analysis of population plays an important role, including the new *Spatial Demography* journal and recent special journal issues on the topic. Second, we revisit the early work on spatial population dynamics, which focuses on extensions of the incorporation of location (place of residence) in the dominant toolkit of demography, including the life table, the demographic accounting equation, the cohort component projection model, and the stable (steady state) population model. Third, we present a framework for how indirect estimation techniques can be utilised to model spatial population dynamics. Finally, we discuss current obstacles and make recommendations for future research.

2 | BACKGROUND

There are currently two demographic-orientated journals where geographic data play an important role. The journal of *Population, Space*

and Place started in September 1995 as the International Journal of Population Geography but, in 2004, changed its title and broadened its scope to "geographical population studies." This journal describes its purpose as follows:

> The scope of the journal is international, covering developed and less developed countries and embracing all the main fields of interest in population studies, including: population and society; fertility, mortality and migration; quantitative and qualitative methods of population analysis; ageing populations; census analysis; spatial demography; population policies; theory and population; population distribution and change; and population and development …. The editors welcome contributions from researchers in all fields of population studies who are interested in geographical issues.

Note that this journal lists spatial demography as one of the areas of interest. It also uses the wider field description of "population studies" rather than "demography."

The second demographically orientated journal is Spatial Demography, which issued its first volume in 2013. Its statement on the journal's scope is as follows:

> Spatial Demography focuses on the spatial analysis of demographic processes. This cross-disciplinary work involves modern demographic data visualization, enhanced geo-referenced data availability, and spatial statistics, facilitated through full color graphics, motion video tools, and a quick time-to-publication. The journal publishes research articles, essays, research reports, data sources, computing software, teaching notes, and book reviews on a wide range of topics of interest to the social demographer.

Since its first issue (and at the time of this writing), there have been 46 research articles published in this journal. Of these 46 articles, we categorised 26 to be population geography, 8 to be spatial statistics, 5 to be health geography, and 1 to be political geography. Only six articles dealt with core demographic issues of population change with four on urbanisation (Bocquier, 2015; Buettner, 2015; Chandrasekhar & Sharma, 2015; Strozza, Benassi, Ferrara, & Gallo, 2016), one on human development and fertility (Porter, 2017), and one on indirect estimation techniques for international migration (Wilson, 2017). Not one of the articles considered interacting populations across space as described in the next section.

Aside from the two journals above, there are several other journals that publish articles related to spatial analyses of population. For example, the Journal of Regional Science

> … publishes original analytical research at the intersection of economics and quantitative geography …. This includes rigorous methodological contributions and seminal theoretical pieces … in urban and regional research, planning, geography, and the environment. The Journal of Regional Science continues to publish work that advances our understanding of the geographic

dimensions of urban and regional economies, human settlements, and policies related to cities and regions.

Also, the journal Applied Spatial Analysis and Policy describes its rational as

> A geographical perspective has always been crucial to the understanding of the social and physical organisation of the world around us. The techniques of spatial analysis provide a powerful means for the assembly and interpretation of evidence, and thus to address critical questions about issues such as crime and deprivation, immigration and demographic restructuring, retailing activity and employment change, resource management and environmental improvement …. Geographical location is critical in much of this work which extends across a wide range of disciplines including demography, actuarial sciences, statistics, public sector planning, business planning, economics, epidemiology, sociology, social policy, health research, environmental management.

Clearly, all the above journals are targeted towards a wide range of social scientists and practitioners conducting spatial analysis. What is not clear is where spatial demography fits. In Population, Space and Place, it is included as a subdiscipline of interest, whereas the journal of Spatial Demography suggests spatial demography is a more general area of research covering a wide range of social science topics.

In addition to the above journals dedicated to the spatial analysis of population studies, there have been some key initiatives on the topic of spatial demography. In 2005, there was a special issue on spatial demography in the Proceedings of the National Academy of Sciences that included seven papers (Wachter, 2005). In 2007 and 2008, Population Research and Policy Review published 15 papers (across two issues) for a special issue on spatial demography (Voss, 2007). In 2013, Demographic Research published six papers as part of a special issue on spatial demography (Matthews & Parker, 2013). Finally, in 2016, an edited volume entitled, Recapturing space: New middle-range theory in spatial demography, was published by Springer as part of a new book series on spatial demography (Howell, Porter, & Matthews, 2016).

Missing from all of the above works are theories and applications related to dynamic spatial population change that embed cohort and life course mechanisms. Most of the recent literature is more or less in line with traditional population geography and regional science, or what Population, Space and Place terms 'spatial population studies.' Core demographic thinking is clearly absent and so are the foundational ideas laid down by Rogers, Willekens, Wilson, Rees and others starting in the late 1960s (Rogers, 1966, 1968, 1975; Rogers & Willekens, 1976; Willekens & Rogers, 1978; A. G. Wilson, 1974; A. G. Wilson & Rees, 1974a, 1974b, 1975; Woods & Rees, 1986). Voss (2007) attributes spatial demography to Woods (1984). To our knowledge, the term "spatial demographic analysis" was used about 10 years earlier by Rees and Wilson (1973).

At present, knowledge and techniques of spatial demography have to come from a wide array of sources. Some notable works include Rogers' (1975, 1995) texts on multiregional mathematical demography, Woods and Rees' (1986) "population structures and models," Plane and Rogerson's (1994) "geographical analysis of population," Smith, Tayman, and Swanson's (2001) "state and local population projections," Stillwell and Clarke's (2011) "population dynamics and projection methods," and Swanson and Tayman's (2012) "subnational population estimates." Moreover, students often learn about the basics of demography in the context of another (larger) discipline, such as sociology, geography, or economics, which could explain for the general absence of demography in spatial demography.

Age, cohort, and location are central to spatial demography. That is precisely the thinking that led the urban and regional planner Rogers, the geographer Rees, and others to develop spatial demographic accounting and multiregional/spatial demography. Rogers started from mathematical demography (i.e., Keyfitz) and input–output modelling, whereas Rees started from geography and introduced Stone's socio-demographic accounting framework with age as the central variable.

The centrality of age in demography leads to the interest in the life course, and the mechanisms underlying the typical age profiles of fertility, mortality, and migration. The life course links the empirical age regularities observed in demographic processes and the explanatory factors producing and changing them over time. The life course perspective also channels the effects of historical factors (and contextual factors in general) on the processes causing population change. These factors generate cohort effects and explain the centrality of them in demography (since Ryder, 1965) and the practice of stratification of populations along cohort lines in order to describe, explain, and predict population change.

3 | FOUNDATIONS OF SPATIAL DEMOGRAPHY

3.1 | Introduction

Life course transitions include, for example, transitions from being single to being married, from being employed to being unemployed, from being in school to having graduated, from living in region i to living in region j, and from being alive to being dead. Work in multistate demographic analysis has produced a generalisation of classical demographic techniques that unifies most of the methods for dealing with transitions between multiple states of existence (Land & Rogers, 1982; Willekens, 2014: Chapter 7). Part of this work has been to demonstrate how multiple decrement mortality tables, tables of working life, nuptiality tables, tables of educational life, and multiregional life tables are all members of a general class of increment–decrement life tables called *multistate life tables*. Similarly, projections of populations classified by multiple states of existence can be carried out using a common methodology of *multistate projection*, in which the core model of the population dynamics is either a generalisation of the continuous age–time model of Lotka or the discrete age–time model of Leslie.

In spatial demography, the timing and location of demographic outcomes are linked to transitions in the individual life course. For instance, a divorce or job loss in one place may trigger migration to another place, or conversely, a migration may result in family separation or unemployment in the destination location. The distinction between multistate demography and multiregional demography is largely dependent upon whether the life course transitions focus on movements between places or between other life states. Of course, one may be interested in the relationships between spatial transitions and other social or economic transitions people make in their life course. For instance, one might want to study the likelihood of unemployment for a person who migrates to a particular region, relative to someone who does not migrate, or to someone who migrates to another region in the system.

Consider the four hypothetical populations presented in Figure 1. Traditional demography would treat each population independently from each other and compare their patterns. We refer to this treatment of spatial units as uniregional demography. Early developments in spatial demography (multiregional demography) connected the four populations through the processes of origin–destination-specific migration and allowed the populations to simultaneously evolve over time. This analytical framework greatly enhanced our understanding for how subnational populations interact and change over time. And it is this perspective that we believe is central and missing from the recent literature on spatial demography.

In the next subsection, we review the basic aspects of multiregional life tables and projections, which form the analytical foundation for studying spatial demographic change. Here, location is an essential attribute of individuals and that attribute changes during the life course. The multiregional life table describes a population in which children are born in different regions and may leave the region at any age, may later move on to another region, or return to the region of birth. Death may occur at any age and in any region. For example, consider a cohort of rural-born babies and a cohort of urban-born babies. A rural youth might migrate to an urban area at a given age to go to school or to join the urban labour force; he or she might return several years later as an adult having married an urban-born; if unsuccessful in entering the rural labour market, he or she might decide to migrate once again, raising his or her children in yet another urban or rural region. This framework allows one to analyse the duration of time spent in each region, controlling for the initial conditions in life. Combining these calculations with information regarding the spatial nature of the population of interest provides analysts with much in terms of explaining the evolutions and spatial interactions of regional populations over time.

The natural starting point for thinking about population change is the probability that an individual born or living in a given region will survive for a given number of years, stay in the region, migrate to another region, and return later or move to on to other regions. By way of illustration, consider two regions, urban (U) and rural (R). An individual of exact age x at time t was born at $t−x$, denoted by $B(t−x)$. The population change can be described by birth cohort or by age. The two approaches are illustrated here. The urban population aged x at t depends on the number of urban-born and rural-
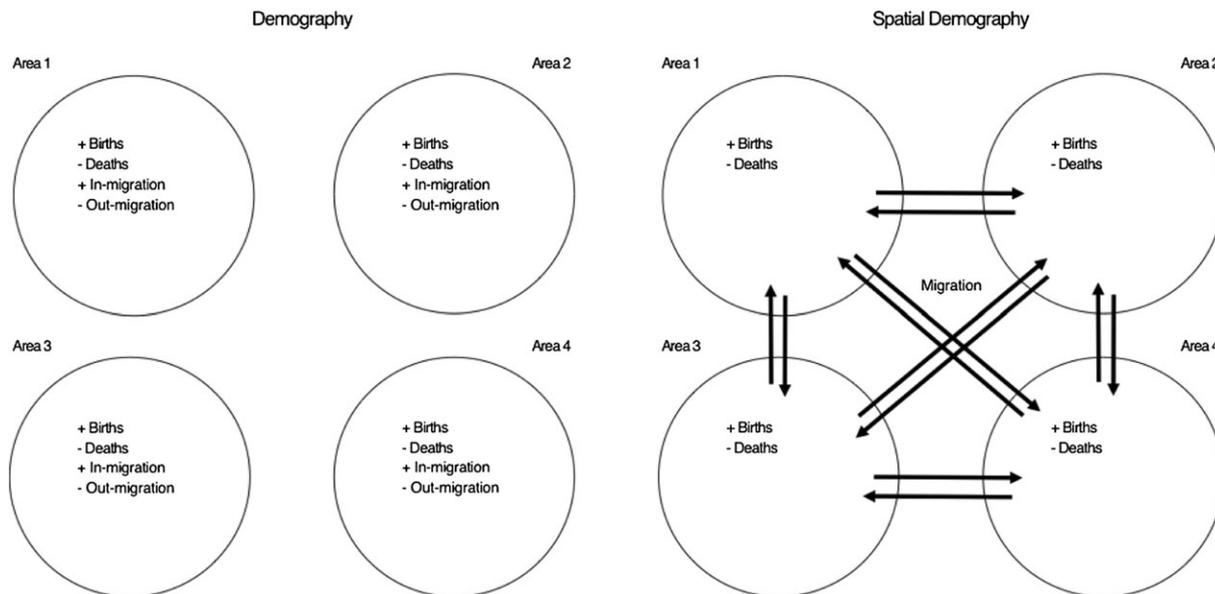
FIGURE 1 Uniregional (left) and multiregional (right) demography

born children and on their survival and migration between birth and age x:

$$K_U(x,t) = B_U(t{-}x)\ _U l_U(x, t{-}x) + B_R(t{-}x)\ _R l_U(x, t{-}x), \quad (1)$$

where $K_i(x,t)$ denotes the population in area i, at time t and age x, and $_j l_i(x)$ denotes the probability that a child born in area j at time $t - x$ is alive and residing in area i at age x. A child may have migrated frequently but is living in U at exact age x at time t.

A similar equation to the one above can be written for the rural population aged x at t. In this equation, only two locations are considered for each x-year old: place of birth and place of residence at age x. Additional locations may be considered during the life course. Consider an age and time interval of h years. The urban population at age $x + h$ and time $t + h$ may be expressed as

$$K_U(x+h, t+h) = K_U(x,t)\ p_{UU}(x,t) + K_R(x,t)\ p_{RU}(x,t), \quad (2)$$

where $p_{ki}(x,t)$ is the probability that an individual residing in area k at age x survives and resides in area i at age $x + h$ at time $t + h$. A similar equation may be written for the rural population at age $x + h$ at time $t + h$.

Extensions of these models to multiple locations (systems of regions) are briefly presented in the Sections 3.2 to 3.4. A system of regions requires multiple equations as those given above and, because of migration is origin- and destination-specific, the equations must be solved simultaneously. Matrix algebra offers convenient and efficient methods for solving systems of equations. The life table is briefly covered in Section 3.2 and population projection in Section 3.3. Section 3.4 addresses a further extension, where a migration is included between the system of regions and areas outside the system. A system of regions that interacts with other systems is an open system. Systems that do not are closed systems.

3.2 | Multiregional life tables

The life table is a central concept in demography. Its use to express the facts of mortality in terms of survival probabilities and their combined impact on the lives of a cohort of people born at the same moment has been so successful that, in the words of Keyfitz (1968: 3), "we are incapable of thinking of population change and mortality from any other starting point." The natural starting point for thinking about spatial population change is the multiregional life table, its theoretical derivation, and its empirical calculation. Probabilities of surviving and migration are central to the multiregional life table. They are derived from death rate and migration rates, estimated from data.

Rogers (1975, 1995) and colleagues have shown that the rates and probabilities can be combined in matrices, which is the starting point of the application of the mathematical theory of matrices. Let $M(x)$ denote the matrix of death rates and migration rates within the age interval x to $x + h$. The definition and configuration of the matrix are described by Rogers and need not concern us here. Two types of life tables are usually distinguished: a cohort life table, which considers as input death and migration rates that vary with age and in time, and period life tables, in which the rates vary with age but not in time. In the period life table, we can derive

$$\left[I + \frac{h}{2} M(x)\right] l(x+h) = \left[I - \frac{h}{2} M(x)\right] l(x), \quad (3)$$

whence

$$P(x) = \left[I + \frac{h}{2} M(x)\right]^{-1} \left[I - \frac{h}{2} M(x)\right]. \quad (4)$$

Those familiar with uniregional life-table construction methods will recognise in Equation (4) the conventional formula for deriving life-table probabilities from observed rates (Rogers & Ledent, 1976). The only difference in the multiregional version is that matrices appear in place of scalars.

3.3 | Multiregional projection models

An important and fundamental application of the survivorship probabilities and proportions found in a multiregional life table is population projection. Multiregional projection models are of two kinds: continuous age–time Lotka models and discrete age–time Leslie models. Moreover, migration between the system of regions and other population systems may be included in the projection models.

A *continuous age-time model* of a closed single-sex population may be defined for a multiregional system by means of a straightforward generalisation of the classical Lotka model. Beginning with the number of female births in each region, $B_j(t)$ say, we note that women aged x to $x+dx$ in state i at time t are survivors of those born $x-dx$ to x years ago and now living in region i, that is, $\sum_{j=1}^{r} B_j(t-x)_j l_i(x) dx$, where $x \leq t$. At time t, these women give birth to $[\sum_{j=1}^{r} B_j(t-x)_j l_i(x)] m_i(x) dx$ children per year while in region i. Here, $_j l_i(x)$ denotes the probability that a baby girl born in region j will survive to age x in region i, and $m_i(x)$ is the annual rate of female childbearing amongst women aged x to $x+dx$ in region i. Integrating this last expression over all ages x and focusing on the population at times beyond the last age of childbearing gives the homogeneous equation system

$$B(t) = \int_0^\beta m(x) l(x) B(t-x) dx. \tag{5}$$

The *discrete age-time model* of multiregional demographic growth is expressed by means of a matrix operation of the population projection process: a multiregional population set out as a vector is multiplied by a growth matrix that projects that population forward through time. The projection calculates the region-specific and age-specific survivors of a multiregional population of a given sex and adds to this total the new births that survive to the end of the unit time interval. This process may be described by the matrix model

$$K(t+1) = G(t) K(t), \tag{6}$$

where the vector $K(t)$ sets out the multiregional population disaggregated by age and region, and the projection matrix $G(t)$ is composed of zeros and elements that represent the various time-varying age-specific and region-specific components of population change. If G does not vary in time, Equation (6) stabilises when G is raised to successively higher powers. The asymptotic properties of the projection Equation (6) have been extensively studied in mathematical demography. It describes the dynamic equilibrium that is fully characterised by the rates of fertility, mortality, and migration in a base period and that emerges in the long term, when the effects of the initial age composition and spatial distribution of the population is phased out. The equilibrium population is used to separate the effects on population growth and distribution of demographic rates (rate effect) and population structure (composition effect). This body of theory draws on the properties of matrices with nonnegative elements and, in particular, on the Perron–Frobenius theorem. Thus, concepts such as the *stable equivalent population* can be used as a basis for analysing the relative importance and long-term potential consequences of current demographic processes across spatial units.

The stable equivalent population is the total which, if distributed according to the stable vector K_1, would ultimately grow at the same rate as the observed $K(0)$ projected by the projection matrix as $G^n K(0)$. The ultimate growth ratio is the dominant eigenvalue of the projection matrix G. The associated right eigenvector, K_1, defines the stable population across ages and states. The left eigenvector, V_1^T, describes the reproductive potential of the multiregional population. The product $V_1^T K(0)$ is known as the *total reproductive value* of the initial population (Rogers & Willekens, 1978), a notion first set out by Fisher (1929: 27).

Implicit in every multiregional projection matrix is a stable distribution across ages and regions, expressible in terms of age compositions and state shares. Deviations from these compositions and shares, in the initial age-by-region distribution, ultimately disappear, but in the short to medium run, they create fluctuations and disturbances in age profiles and in allocations over regions (Liaw, 1986).

3.4 | The spatial dynamics of multiregional population models "open" to other population systems

The ages at which immigrants are admitted can be shown to make a significant difference in the ultimate population size and spatial distribution and so can the region of entry. Multiregional versions of the life expectancy and the reproductive value may be used to assess these impacts. For example, Rogers (1990a: 315) shows that if in the closed model described by Equation (6) with constant projection matrix, we add a count of immigrants from another population system, X, on the assumption of unchanging rates and a fixed immigration stream one finds that the population that will result in t years is

$$K(t) = G^t K(0) + (I-G)^{-1}(I-G^t) X. \tag{7}$$

Here, international migrants exiting the system are included as rates or probabilities in the G matrix and treated as attritions, similar to deaths. And, if fertility is below replacement level, then the stable equivalent population can be shown to be

$$Q = (I-G)^{-1} X, \tag{8}$$

where Q is the stable (in this case, stationary) population equivalent of the observed population, K(0). Studying how regional populations are changing in relation to multiregional population stability provides an indication of the speed and potential long-term consequences of recent demographic events. By doing so, we can also assess how efficient populations are redistributing across space. In reality, we expect the demographic rates to change and respond to various economic and social processes over time. However, it is useful to have a benchmark for which to compare such changes.

4 | ISSUES IN APPLIED MULTIREGIONAL DEMOGRAPHY

4.1 | Data and accounts

Empirical studies in multiregional demography often begin with data set out in tabular form, which describe changes in *stocks* that have

occurred over two or more points in time. These changes arise as a consequence of increments and decrements associated with *events*, such as births and deaths, and with *flows* of individuals between different regions.

When all of the appropriate elements in such tables have been filled in with numbers, they generally are referred to as *accounts*. And when, as is often the case, some data are unavailable, ingenuity, and indirect estimation are used to supply the missing entries. Prominent amongst such techniques are various row and column balancing methods that have been successfully implemented in economics (input–output matrices), transportation planning (origin–destination traffic flows), and statistics (contingency tables).

The idea of arranging monetary transactions in a system of interlocking statements, in which total inflows are forced to equal total outflows, is a familiar habit of thought in economics. The utility of imposing a similar habit to the inescapable accounting interrelationships that arise in spatial demographic data is just as important.

4.2 | Movements and transitions in multiregional life tables

During the course of a year or some such fixed interval of time, a number of individuals change their current region of residence. A move out of a region of residence is an event, a separation. A mover is an individual who has made a move at least once during a given interval of time. A migrant, on the other hand, is an individual who at the end of a given time interval no longer inhabits the same region of residence as at the start of the interval. The migrant has made a transition from one region to the other. The act of separation from one region is linked with an addition to another. Thus, paradoxically, a multiple mover may be a nonmigrant by this definition; if, for example, a particular mover returns to the initial region of residence before the end of the unit time interval, no 'migration' is registered.

The crux of the life-table construction problem is the estimation of age-specific survival probability transition matrices, P(x), by use of data either on interstate moves or on interstate transitions (i.e., "Option 1" and "Option 2," respectively, in Rogers, 1975, 1995). Because the data on multistate flows can come in the form of move counts or people counts, the methods used must be specific to each kind of data (Rees & Willekens, 1986). Irrespective of the form of the data, however, no statements about probabilities can be made without a conversion of 'moves' information to 'people' information at some point in the analysis.

4.3 | Origin dependence in migration flows

Studies of the spatial patterns of directional migration flows have found that individuals who have moved before are more likely to do so again and to destinations that they have lived in earlier in life, especially their birthplace (DaVanzo, 1983; Eldridge, 1965; Newbold, 1997; Rogers, 1990b; Waldorf, 1995). The most important consequence is that the probabilities of return migration to their region of birth are significantly higher than those of the average individual, and their age-specific migration rates differ in the age profile.

If birthplace specificity is introduced, then each age-specific, origin–destination-specific flow can be expressed as the sum of three distinct categories of migrants: (a) persons leaving their region of birth: primary migrants; (b) persons returning to their region of birth: return migrants; and (c) persons migrating neither from nor to their region of birth: onward migrants. The motivations and patterns associated with return migrants typically are quite different from those of non-return migrants.

Because the migration patterns of return migrants are significantly different from those of non-return migrants, the incorporation of such differentials in spatial processes can sometimes produce unexpected results. For example, such results can identify the reason for a strongly positive correlation between rates of out-migration and in-migration across different regions. Moreover, they can answer the question whether the elderly are more likely to return home (Rogers, 2015: Chapter 4). Unfortunately, these data are not easy to obtain for many countries because of the reluctance of official statistics agencies to publish very detailed data matrices due to the risk of disclosure of individuals, and sampled data are usually not of sufficient size to capture the relatively small numbers of people moving between regions.

4.4 | Shrinking very large population projection models

An increasing number of demographers find themselves in the somewhat frustrating position of being asked to provide accurate population projections at very fine levels of detail with resources that are scarcely adequate for carrying out such projections at much more aggregate levels of resolution. Prominent amongst them are those called upon to produce consistent projections of regional populations disaggregated by location, age, sex, ethnicity, and such indicators of class and welfare as employment category and income. Imagine the daunting task faced by a demographer of projecting, in a consistent manner and in such detail, the future populations of the nearly 300 metropolitan areas of the contemporary United States. To produce consistent projections means avoiding net migration rates and uniregional projections for each metropolitan area. How best to "shrink" the huge multiregional model? What approaches are available to reduce the dimensionality of the fundamental problem? Fortunately, there are some examples of very large and detailed population projections from which we can draw experiences from, notably Wilson (2011) for local areas in New South Wales, Australia, Rees, Wohland, Norman, and Boden (2012) for ethnic projections across local authorities in the United Kingdom, and Lutz, Butz, and KC (2014) for human capital projections for all countries in the world. See also Rogers (1976), Rees (1997), Wilson and Bell (2004), and Wilson (2016).

Because of the staggering data and computational requirements of very large population projection models, there is a fundamental need for (a) improved methods of shrinking such models and for (b) indirectly estimating some of the necessary detailed data. The former task leads one to focus on aggregation and partitioning, or more appropriately, decomposition methods. The latter task may be addressed by considering indirect estimation methods (see following section).

The notion that it might be useful to model different parts of a large system at different levels of detail received its first mathematical treatment more than a half century ago in a seminal paper by Simon and Ando (1961) published in *Econometrica*. This article suggests the following simple method for shrinking large-scale population projection models. One begins by partitioning a multiregional system into its constituent single regions and projecting their growth and change as if they were independent, closed population subsystems undisturbed by migration. The first stage, therefore, corresponds to a uniregional decomposition with zero net migration. The second stage involves suppressing all age-specific details and projecting the multiregional population by using an aggregate components-of-change model. The results of the latter stage determine the total multiregional population and its spatial distribution: the results of the first stage define the individual regional age compositions. In this way, within-subsystem interactions (i.e., changes in age structure) are modelled at a fine level of detail, whereas between-subsystem interactions (i.e., changes in spatial structure) are modelled at a coarse level of detail. If the original multiregional system is sufficiently close to being nearly decomposable, the approximate (two-stage) projection should produce a reasonably accurate multiregional population projection. This was confirmed by Wilson and Bell (2004, p. 157) in examining different migration specifications for 10 projection models; they found that "the processes of partitioning and aggregation, designed to reduce the size of the multiregional matrix, result in very little difference from the standard multiregional model."

5 | TOOLS FOR INDIRECT ESTIMATION IN SPATIAL DEMOGRAPHY

Another aspect of demography that is needed for studying spatial population dynamics is indirect estimation. As population models become larger and more detailed, techniques for simplifying the estimation process and dealing with poor data become more important (Rees, 1997). Methods for indirect estimation are required when data are incomplete, inadequate, or missing. They are particularly useful when analysing population change for small areas or for subpopulations (e.g., ethnic population change). In this section, we focus on two general methods used widely in the social sciences: spatial interaction models and model age schedules of demographic events. The former are used to describe the spatial structure of flows; the latter to describe the age structure of events and flows.

5.1 | Spatial interaction models of migration

When populations interact across space, a truer sense of population change and the processes underlying population redistribution is obtained. In spatial demography, migration flows are the main mechanism for interaction and therefore have been the subject of extensive modelling research. The gravity model is the oldest spatial interaction model (Fotheringham, Brunsdon, & Charton, 2000: Chapter 9). It is based on the premise that the interaction between two places is proportional to the sizes of the places (in terms of population and/or economic activity) and inversely related to (some function of) the distance between the places. Spatial interaction models may be applied to any flow process that involves two or more locations, however, for the purposes of this paper, we focus on the migration process. As part of one of the most ambitious studies of internal migration ever taken, Stillwell et al. (2016) fitted spatial interaction models to a series of migration flow tables representing 105 countries to study the effect of distance while controlling for modifiable area unit problem. In another ambitious study, Cohen, Roig, Reuman, and GoGwilt (2008) fitted a gravity model, specified as a generalised linear model, to estimate flows of international migration amongst all countries in the world.

In the early 1980s, Flowerdew and Aitkin (1982) argued for (a) linking spatial interactions to exposure, (b) accounting for the uncertainty in migration flows, and (c) calibrating the gravity model using techniques of statistical inference. They argued that the number of persons migrating between two places during a given period can be approximated by a Poisson random variable, with the possible values described by a Poisson distribution. Further, they showed that the gravity model can be considered in the family of the generalised linear models. A distinguishing feature of members of that family of models is that a transformation of the dependent variable can be written as a linear function of the independent variables. In the Poisson model, the transformation is the logarithmic transformation. The Poisson regression model is a log-linear model. If the variables are discrete, the log-linear model can be estimated using statistical techniques of discrete multivariate analysis, better known as log-linear analysis and categorical data analysis (Bishop, Fienberg, & Holland, 1975). Willekens (1983) showed the similarity between these techniques and calibration methods used traditionally for estimating gravity models.

Spatial demography involves the modelling of geo-referenced population dynamics. To overcome limitations in available origin–destination migration flows or missing data, spatial interaction models may be integrated in the multiregional life table and the multiregional projection model, highlighted in the previous section. The natural starting point for thinking about this is the migration rate by age, origin, and destination. In a set of migration rates by age, origin, and destination, two core structures can be identified: the spatial structure and the age structure. Spatial interaction models focus on the spatial structure; models of migration age schedules (described in the next section) focus on the age structure.

Let $N_{ij}(x)$ denote the number of persons of age x that migrate from area i to area j in a given time interval. Age x may be measured at time of migration or at the beginning of the interval. The latter case is when migration data are available by year of birth, for example, in census data. $N_{ij}(x)$ is a random variable. The expected value of $N_{ij}(x)$, $E[N_{ij}(x)]$, and its variance, $Var[N_{ij}(x)]$, is estimated from migration counts. The data are count data and assumed to be outcomes of an underlying counting process, which is a particular type of stochastic process (Aalen, Borgan, & Gjessing, 2008). Contrary to more traditional approaches, observed migration counts are not modelled directly. The underlying stochastic process is modelled instead, and the observed event counts are used to estimate the parameters of the process model. This approach bridges the divide between demographic modelling, probability theory, and statistical inference. The Poisson process assumes that all migrations are independent and

follow the same probability distribution. This process leads to the Poisson regression model mentioned earlier. The random variable $N_{ij}(x)$ follows a Poisson distribution. The probability of exactly $n_{ij}(x)$ migrations during an interval of length h is

$$\Pr\{N_{ij}(x) = n_{ij}(x)\} = \frac{h\xi_{ij}(x)}{n_{ij}(x)!}\exp\left[-h\xi_{ij}(x)\right], \quad (9)$$

where $\xi_{ij}(x)$ is the parameter of the Poisson distribution. The expected value and the variance of events in a unit interval are equal to $\xi_{ij}(x)$. The parameter $\xi_{ij}(x)$ depends on the migration rate and total duration of exposure during the time interval by the population at risk of migrating:

$$\lambda_{ij}(x) = PY_i(x)\mu_{ij}(x), \quad (10)$$

with $PY_i(x)$ the exposure time during a unit interval and $\mu_{ij}(x)$ the rate at which a person of age x living in i migrates to j. Note that residents of i only are at risk of migrating to j. In most applications, it is assumed that the exposure time is fixed and does not depend on the randomness in migrations. In that case, $PY_i(x)$ is treated as an offset in the Poisson regression model.

Poisson regression models can be formulated with $\xi_{ij}(x)$ or $\mu_{ij}(x)$ as the dependent variable. In spatial interaction models, $\xi_{ij}(x)$ is usually used. For instance, the gravity model can be written as

$$\xi_{ij}(x) = k(x)\alpha_i(x)\beta_j(x)\exp\left[-c_{ij}(x)\right], \quad (11)$$

where $\gamma_{ij}(x) = \exp[-c_{ij}(x)]$ is an age-specific spatial friction or distance deterrence factor related to the geographic and cultural distance between i and j. The friction factor may be replaced by a preliminary estimate of the migration flow, for example, a flow observed at some prior date. The migration rate is $\mu_{ij}(x) = \xi_{ij}(x)/PY_i(x)$, where exposure time is determined separately (e.g., as the product of the midperiod population and the length of the period).

The gravity model can be written as a log-linear model:

$$\ln\left[\xi_{ij}(x)\right] = u + u_i^O + u_j^D + u_{ij}^{OD}, \quad (12)$$

where u is the overall effect, u_i^O is the effect associated with origin i, u_j^D is the effect associated with destination j, and u_{ij}^{OD} is the interaction effect between origin and destination. For a review of spatial interaction models of migration, refer to Willekens (2008) and Rogers, Little, and Raymer (2010). The gravity model can also be extended to include other geographic factors besides distance that influence migration flows, for example, area-specific unit sizes, socio-economic factors, contiguity, and population density. For further details and applications of spatial interaction models, refer to Stillwell and Congdon (1991) and Stillwell, Duke-Williams and Dennett (2010).

5.2 | Model schedules of demographic events and flows

The age-specific patterns of fertility, mortality, and migration exhibit strong regularities that can be used to overcome data limitations in a wide array of settings, including small populations where the data may be sparse. Coale and Trussell (1974), Heligman and Pollard

(1980), and Rogers and Castro (1981) developed parameter-based model schedules for estimating age-specific fertility, mortality, and migration, respectively. The relatively stable shapes of age-specific demographic events provide demographers with the possibility to simplify their underlying assumptions and methods for estimation or projection. These age profiles are reflective of life course transitions that populations experience and the patterns may vary depending on spatial context (Bernard, Bell, & Charles-Edwards, 2014).

For example, the research by Rogers and Castro (1981) demonstrated the persistent and strong regularities in the age patterns of internal migration over time and across space. The general shape of the propensity to migrate begins with a downward slope from early childhood to middle childhood, followed by a sharp "labour force" increase in the young adult ages, and finally, a general downslope to the oldest ages with some instances of a second "retirement" peak centred around ages 60–65 years. These same regularities are also found in patterns of international migration (see,e.g., Wiśniowski, Forster, Smith, Bijak, & Raymer, 2016).

Not all age profiles of migration are the same, but most have a downward sloping curve for young children and a single labour force peak. The more complicated shapes that include, for example, a student migration, a second retirement peak, or an upward slope in the oldest age groups, may be explained by the attributes found at the origin or destination (Warnes, 1992; Wilson, 2010). For example, the emigration flows of nationals usually have younger labour force peaks and fewer children than the corresponding flows of immigrant nationals (i.e., return migrants). Locations dominated by universities have very sharp young adult peaks, whereas those with a range of employment and education activities attract a wider range of young adults (Plane & Heins, 2003). If the location attributes are known, then the age patterns of migration may be inferred. If the age patterns are not known or if they are believed to differ from the parametric specification of the model migration schedule, then cubic splines or kernel regressions may be used to smooth and identify age regularities present in the data (Bernard & Bell, 2015).

6 | DISCUSSION AND FUTURE RESEARCH AGENDA

In this paper, we argue that spatial demography currently lacks a unifying core for research enquiry and propose that multiregional models be included as a central part for such a core. The reason is threefold. First, multiregional methods incorporate spatial interaction and thus bring together population flows and stocks. They focus on the populations "at risk" of experiencing events and therefore avoid reference to net flows. Second, multiregional models track individuals across several changes of residence across space and allow the disaggregation of current or future stocks and flows of individuals by their previous places of residence. Third, multiregional demographic methods enhance our understanding of regional mortality, fertility, and migration and how populations are interconnected across space.

When subpopulations and their demographic components of change are interconnected by directional migration flows, important demographic questions that are inherently spatial can be addressed.

For example, do migrants from rural areas in cities have higher or lower fertility? How many of the migrants who arrived in the past 10 years are still in the country? And how many can be expected to remain for retirement? And if fertility levels remain low in cities and high in rural areas, what might the future population distribution be for different migration scenarios? These types of questions place demography, or the study of population change, at the core of spatial thinking and enquiry.

A framework for spatial demography, as proposed in this paper, has been developed in multiregional mathematical demography. However, there are plenty of areas where research is needed to improve the framework. In particular, there is a need to integrate recent developments in stochastic processes and spatial dependency modelling. Although fixed-rate multiregional models have been useful for understanding the mechanisms of interacting subpopulations and the implications of particular rates within a system, they are often considered unrealistic or practical for planning. Research is needed to generalise the model for dynamic rates and probabilistic perspectives utilising time series models (e.g., Chatfield, 2004). Research is also needed to integrate spatial analysis techniques (e.g., Fotheringham et al., 2000; Oyana & Margai, 2016). As noted by Swanson and Tayman (2012), this is a major issue especially when dealing with small geographic areas. Finally, there is a real need for investing in models to estimate and forecast age-specific international migration flows for inclusion in multiregional model projections (Abel, 2013; Raymer, Wiśniowski, Forster, Smith, & Bijak, 2013; Wiśniowski et al., 2016).

Accounting for the large number of correlations present in spatial demographic data can make spatial demographic models complex and difficult to fit. The correlations include those across ages, cohorts, over time, and space. Moreover, with migration, there are often correlations in the counterflows, that is, migration from i to j is related to migration from j to i. In the population forecasting literature, there has been some progress to account for these correlations. For example, Wiśniowski, Smith, Bijak, Raymer, and Forster (2015) used Bayesian methods to estimate age-specific demographic components of change by extending the well-known Lee and Carter (1992) model for mortality in their population forecasting model for the United Kingdom. They explored correlations across time, sex, and in the components of demographic change, but there were no subnational populations included in the analysis. Raymer, Abel and Rogers (2012) utilised vector autoregressive models to capture the serial autocorrelation and spatial dependency in demographic components across regions in England, but their models did not account for age or sex. Finally, the recent developments in Bayesian methods applied to demographic estimation and projection have much potential for both transparency and ability to both borrow strength across patterns in the data (Bijak & Bryant, 2016; Raftery, Li, Ševčíková, Gerland, & Heilig, 2012).

The multiregional framework also provides a foundation for understanding individual behaviours using microsimulation or agent-based modelling. Here, population dynamics are the outcomes of actions that individuals make. Agent-based models may be used to study population dynamics resulting from decisions individual actors make and the subsequent social processes that form when actors interact and influence each other (e.g., emergence of social networks). In these models, decisions and rules governing social interaction and the resultant diffusion of behaviour replace the empirical rates of migration that are normally used as the main parameters in multiregional demographic models. See Klabunde and Willekens (2016) for a review of agent-based models of migration. A major challenge in developing agent-based models for spatial demography is the operationalisation of decision theories and theories of social interaction and influence. To predict international migration, for example, Willekens (2017) and Klabunde, Zinn, Willekens, and Leuchter (2017) operationalised the well-known theory of planned behaviour (Fishbein & Ajzen, 2010).

The main obstacles limiting a spatial demography perspective include training and data availability. As De Castro (2007) notes, there are very few places in the world that provide courses in both demography and spatial analysis, which limits the "spatial thinking" amongst demographers and the "demographic thinking" amongst spatial analysts. The methodological skills required to do spatial demography include mathematical demography and a range of statistics including regression techniques that account for serial autocorrelation, spatial autocorrelation, and different measurements in the dependent variables (generalised linear models). Moreover, it makes sense to have some training in Bayesian methods or microsimulation to deal with complex interactions or to imbed stochastic processes. The lack of courses and textbooks that pulls these aspects together is clearly something that needs to be addressed.

Data availability or data sparseness is an important issue, especially when the interest is understanding a large number of interacting populations and how they change over time. When disaggregated by age and sex, often the population counts and demographic events become small, which increases the effects of both random behaviour and the chances of disclosure (identification). Many statistical offices are not allowed to release very detailed spatial data for reasons of data confidentiality, and this limits the detail of the analyses. Furthermore, data on age-specific rates of fertility, mortality, and migration (interregional and international) for subnational populations may not be available or measured appropriately for demographic accounting—as the data come from different sources (censuses, vital registers, and administrative sources). Indirect estimation may be used to overcome some of these issues.

Finally, as the size of the spatial system increases so does the complexity. This is most readily observed in the origin–destination tables of interregional migration: a 10 region population system has 90 interactions, a 20 region system has 380 interactions, a 30 region system has 870 interactions, and so on. The good news is that computing power and software to deal with complex data have improved immensely since the advent of multiregional demography and continues to do so.

In conclusion, we believe that the field of spatial demography requires dynamic change amongst interacting populations to be at the core of its thinking and analysis. This focus makes it distinguishable from other spatial social sciences and especially population geography, where the interest is understanding differences in the spatial patterns. The foundations for spatial demography were laid nearly 50 years ago, but they appear to have been forgotten in recent research directed at understanding spatial patterns of population change (Bell, 2015). We hope this article has demonstrated that

multiregional dynamics are a central and unifying theme in spatial demography.

CONFLICT OF INTEREST STATEMENT

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